Answers to Selected Exercises for Chapter 1

Section 1.1 (page 16)

1. $\bar{x} = -1.4$, $s = 2.880972$

3. $\sqrt{5} \approx 2.236067977$

5.
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GIVEN:    the integers n, m and p
          the elements in the matrix A, a_ij
          the elements in the matrix B, b_jk
STEP 1:   for i from 1 to n
STEP 2:       for k from 1 to p
STEP 3:           initialize sum to 0
STEP 4:           for j from 1 to m
                      add a_ij b_jk to sum
                  end
STEP 5:           set c_ik = sum
              end
          end
OUTPUT:   the elements in the matrix C, c_ik
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7. (a)
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GIVEN:    non-zero real number a
          starting approximation x_0
          convergence parameter ε
          maximum number of iterations Nmax
STEP 1:   for iter from 1 to Nmax
STEP 2:       compute x_1 = x_0(2 − a x_0)
STEP 3:       if |x_1 − x_0| < ε, OUTPUT x_1
STEP 4:       copy the value of x_1 to x_0
          end
OUTPUT: "maximum number of iterations has been exceeded"
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(b) $1/37 \approx 0.027027016$

9. (b) $\mathbf{x} \cdot \mathbf{y} = -3$

11. (b) $\int_1^2 dx/x \approx 0.691219891$

15. (a)
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GIVEN:    convergence parameter ε
          maximum number of terms Nmax
STEP 1:   initialize sum to 4 and sign to -1
STEP 2:   for i from 1 to Nmax
STEP 3:       set term = 4/(2i + 1)
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STEP 4:        if term < ε, OUTPUT sum
STEP 5:        replace sum by sum + sign * term
STEP 6:        replace sign by −sign
             end
OUTPUT:      "more terms needed"
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(b) $\pi \approx 3.1391$

17. (b) $\sin(\pi/10) \approx 0.309017054$

Section 1.2 (page 27)

1. (a) $0 + O(1/n^2)$ (b) $0 + O(1/\sqrt{n})$ (c) $0 + O(1/n)$ (d) $3/7 + O(1/n)$
3. $(\sin x^2)/x^2 = 1 + O(x^4)$; $(\sin x)^2/x^2 = 1 + O(x^2)$
7. (a) order $> \alpha$ (b) order $< \alpha$
9. Yes, data suggests $\alpha = 1.618$.
11. $\lambda = 1/4a$
17. (a) $P_3(x) = 1 + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{8}x^3$; $R_3(x) = -\frac{15}{16}x^4(1+\xi)^{-3/2}$
(b) $P_3(1.5) = 1.234375$; $|R_3(1.5)| \le 0.05859375$;
$|P_3(1.5) - \sqrt{1.5}| = 0.009630129$
(c) $-1/4 + O(x)$

Section 1.3 (page 39)

1. In the following table, δ denotes the absolute error and ϵ the relative error.

	Chopping		Rounding	
y	$fl(y)$	Error	$fl(y)$	Error
π	3.141	$\delta = 5.927 \times 10^{-4}$ $\epsilon = 1.886 \times 10^{-4}$	3.142	$\delta = 4.073 \times 10^{-4}$ $\epsilon = 1.297 \times 10^{-4}$
$\sqrt{2}$	1.414	$\delta = 2.136 \times 10^{-4}$ $\epsilon = 1.510 \times 10^{-4}$	1.414	$\delta = 2.136 \times 10^{-4}$ $\epsilon = 1.510 \times 10^{-4}$
$\cos 22°$	0.9271	$\delta = 8.385 \times 10^{-5}$ $\epsilon = 9.044 \times 10^{-5}$	0.9272	$\delta = 1.615 \times 10^{-5}$ $\epsilon = 1.741 \times 10^{-5}$
$\sqrt[3]{9}$	2.080	$\delta = 8.382 \times 10^{-5}$ $\epsilon = 4.030 \times 10^{-5}$	2.080	$\delta = 8.382 \times 10^{-5}$ $\epsilon = 4.030 \times 10^{-5}$

7. (a) $0.03555 < n < 0.03592$ (b) $0.46612 < P < 0.46836$
9. $8.929 < g < 9.864$
11. machine precision $= 2^{-64} \approx 5.421 \times 10^{-20}$,
smallest positive number $= 2^{-16382} \approx 3.362 \times 10^{-4932}$,
largest positive number $= (1 - 2^{-64}) \cdot 2^{16384} \approx 1.190 \times 10^{4932}$.
13. machine precision $= 5 \times 10^{-10}$,
smallest positive number $= 10^{-99}$,
largest positive number $= (1 - 10^{-10}) \cdot 10^{100} \approx 10^{100}$.

15. (a) 2,2 (b) 2.0001, 1.9999 (c) $2 \pm 0.0001 \cdot \mathrm{i}$.

17. (a) $x(t) = t(\frac{2x_0}{\pi} - \cos t)$
(b) $x(t) = t(\frac{2(x_0+\epsilon)}{\pi} - \cos t)$
(c) difference between two solutions grows with time; hence, ill-conditioned.

Section 1.4 (page 50)

1. In the following table, δ denotes the absolute error and ϵ the relative error.

	Chopping		Rounding	
	value	Error	value	Error
(a)	4.931	$\delta = 1.691 \times 10^{-3}$ $\epsilon = 3.427 \times 10^{-4}$	4.933	$\delta = 3.094 \times 10^{-4}$ $\epsilon = 6.272 \times 10^{-5}$
(b)	2.005	$\delta = 2.219 \times 10^{-3}$ $\epsilon = 1.105 \times 10^{-3}$	2.007	$\delta = 2.185 \times 10^{-4}$ $\epsilon = 1.089 \times 10^{-4}$
(c)	5.108	$\delta = 1.599 \times 10^{-3}$ $\epsilon = 3.129 \times 10^{-4}$	5.110	$\delta = 4.011 \times 10^{-4}$ $\epsilon = 7.850 \times 10^{-5}$
(d)	−0.1417	$\delta = 1.284 \times 10^{-4}$ $\epsilon = 9.051 \times 10^{-4}$	−0.1418	$\delta = 2.837 \times 10^{-5}$ $\epsilon = 2.000 \times 10^{-4}$

3. in STEP 1 if $b^2 \approx 4ac$, in STEP 2 if $b \approx -disc$, in STEP 3 if $b/a \approx -root1$.

5. using Taylor series, show that the relative error is roughly $x^3/120$.

7. (a) between 4 and 5 (b) 5 decimal digits are lost
(c) $\sqrt{10002} - \sqrt{10001} = 1/(\sqrt{10002} + \sqrt{10001})$.

9. (b) $f(x) = 2\sin^2(x/2)$.

11. (b) using Taylor series, $f(x) \approx \frac{1}{3}x^3 - \frac{7}{24}x^4$.

13. (b) use $g(x)$ to avoid cancellation in $1 - \sin x$.
(c) use $f(x)$ to avoid cancellation in $1 + \sin x$.

15. (a) $x_1 = 1, x_2 = 2, x_3 = -1$ (b) system is inconsistent.