Answers to Selected Exercises for Chapter 3

Section 3.0 (page 148)

1. (a) $\begin{bmatrix} 6 & 1 & 8 \\ 6 & -1 & 6 \end{bmatrix}$ (b) operation not defined because matrices do not have same dimensions

3. (a) $\begin{bmatrix} 8 & -4 & 22 \\ 1 & -3 & 4 \\ -6 & -2 & -14 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -9 \\ 18 & -16 \end{bmatrix}$

5. (a) $\begin{bmatrix} 11 & 3 \\ -5 & -25 \\ 21 & -17 \end{bmatrix}$ (b) operation not defined because the number of columns in C is not equal to the number of rows in B

7. (a) 6 (b) operation not defined because A is not a square matrix

9. (a) $\begin{bmatrix} 18 & 6 \\ 17 & -5 \\ 4 & -30 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & 1 \\ 0 & 5 & 7 \end{bmatrix}$

11. No. If A is and $n \times m$ matrix, then A^T is an $m \times n$ matrix. With $n \neq m$, an $n \times m$ matrix can never be equal to an $m \times n$ matrix.

Section 3.1 (page 157)

1. $\mathbf{x} = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}^T$
3. $\mathbf{x} = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}^T$
5. $\mathbf{x} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$
7. $n^2 - n$ additional arithmetic operations
9. (a) The number of arithmetic operations is given by

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[1 + \sum_{k=i+1}^{2n} 2\right] + \sum_{i=2}^{n} \sum_{j=1}^{i-1} \left[1 + \sum_{k=i+1}^{2n} 2\right] + n^2$$

(b) The number of arithmetic operations is given by

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[1 + \sum_{k=i+1}^{n+i-1} 2\right] + \sum_{i=2}^{n} \sum_{j=1}^{i-1} \left[1 + \sum_{k=i+1}^{n+i-1} 2\right] + n^2$$

11. (a) $\mathbf{x} = \begin{bmatrix} 3.7 & -1.9 & -7.0 \end{bmatrix}^T$
(b) $\mathbf{x} = \begin{bmatrix} 3.707415 & -1.914830 & -7.037074 \end{bmatrix}^T$
(c) $\mathbf{x} = \begin{bmatrix} 3.679 & -1.873 & -6.94 \end{bmatrix}^T$

13. $I_1 = 6.88, I_2 = 4.80, I_3 = 2.08$

15. $\mathbf{g} = \begin{bmatrix} -0.00472 & -3.84552 & -0.71974 & -0.12518 \end{bmatrix}^T$

Section 3.2 (page 168)

1. (a) no pivoting: row 2, column 1; partial pivoting: row 2, column 1; scaled partial pivoting: row 2, column 1
(b) no pivoting: row 1, column 1; partial pivoting: row 4, column 1; scaled partial pivoting: row 1, column 1
(c) no pivoting: row 1, column 1; partial pivoting: row 2, column 1; scaled partial pivoting: row 3, column 1
(d) no pivoting: row 1, column 1; partial pivoting: row 1, column 1; scaled partial pivoting: row 2, column 1
(e) no pivoting: row 1, column 1; partial pivoting: row 3, column 1; scaled partial pivoting: row 4, column 1

3. no pivoting

$$\left[\begin{array}{cccc|c} 3 & -1 & 3 & 1 & 6 \\ 0 & 2 & 3 & -4 & 1 \\ 0 & -4 & 2 & 9 & 7 \\ 0 & 16 & -24 & -43 & -51 \end{array}\right]$$

partial pivoting

$$\left[\begin{array}{cccc|c} 0 & -\frac{2}{3} & 1 & \frac{43}{24} & \frac{17}{8} \\ 0 & \frac{2}{3} & 5 & -\frac{5}{12} & \frac{21}{4} \\ 0 & -\frac{4}{3} & -2 & \frac{11}{6} & -\frac{3}{2} \\ 72 & 8 & 48 & -19 & 93 \end{array}\right], \quad \mathbf{r} = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

scaled partial pivoting

$$\left[\begin{array}{cccc|c} 3 & -1 & 3 & 1 & 6 \\ 0 & 2 & 3 & -4 & 1 \\ 0 & -4 & 2 & 9 & 7 \\ 0 & 16 & -24 & -43 & -51 \end{array}\right], \quad \mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} 3 \\ 9 \\ 12 \\ 72 \end{bmatrix}$$

5. no pivoting

$$\left[\begin{array}{ccc|c} 0.25 & 0.35 & 0.15 & 0.60 \\ 0 & -0.08 & 0.13 & 0.42 \\ 0 & -0.01 & 0.16 & 0.34 \end{array}\right]$$

partial pivoting

$$\left[\begin{array}{ccc|c} 0.25 & 0.35 & 0.15 & 0.60 \\ 0 & -0.08 & 0.13 & 0.42 \\ 0 & -0.01 & 0.16 & 0.34 \end{array}\right], \quad \mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

scaled partial pivoting

$$\left[\begin{array}{ccc|c} 0 & 0.10 & -0.1625 & -0.525 \\ 0.20 & 0.20 & 0.25 & 0.90 \\ 0 & 0.05 & 0.0625 & 0.025 \end{array}\right], \quad \mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} 0.35 \\ 0.25 \\ 0.25 \end{bmatrix}$$

7. $\mathbf{x} = \begin{bmatrix} 2 & -3 & 1 \end{bmatrix}^T$

(a)

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & -4 \\ 4 & 1 & 4 & 9 \\ 3 & 4 & 6 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & \frac{5}{2} & -1 & -\frac{17}{2} \\ 4 & 1 & 4 & 9 \\ 0 & \frac{13}{4} & 3 & -\frac{27}{4} \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 0 & -\frac{43}{13} & -\frac{43}{13} \\ 4 & 1 & 4 & 9 \\ 0 & \frac{13}{4} & 3 & -\frac{27}{4} \end{array}\right]$$

$$\mathbf{r} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}^T$$

(b)

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & -4 \\ 4 & 1 & 4 & 9 \\ 3 & 4 & 6 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & \frac{5}{2} & -1 & -\frac{17}{2} \\ 4 & 1 & 4 & 9 \\ 0 & \frac{13}{4} & 3 & -\frac{27}{4} \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & \frac{5}{2} & -1 & -\frac{17}{2} \\ 4 & 1 & 4 & 9 \\ 0 & 0 & \frac{43}{10} & \frac{43}{10} \end{array}\right]$$

$$\mathbf{r} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}^T$$

$$\mathbf{s} = \begin{bmatrix} 3 & 4 & 6 \end{bmatrix}^T$$

9. $\mathbf{x} = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}^T$

(a)

$$\left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 1 & 2 & -2 & 7 \\ 2 & 5 & 4 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 0 & -\frac{1}{2} & -4 & \frac{15}{2} \\ 2 & 5 & 4 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 0 & 0 & -\frac{23}{6} & \frac{23}{3} \\ 2 & 5 & 4 & -1 \end{array}\right]$$

$$\mathbf{r} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}^T$$

(b)

$$\left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 1 & 2 & -2 & 7 \\ 2 & 5 & 4 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 1 & 2 & -2 & 7 \\ 0 & 1 & 8 & -15 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 1 & 2 & -2 & 7 \\ 0 & 0 & \frac{23}{3} & -\frac{46}{3} \end{array}\right]$$

$$\mathbf{r} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}^T$$

$$\mathbf{s} = \begin{bmatrix} 3 & 2 & 5 \end{bmatrix}^T$$

11. $\mathbf{x} = \begin{bmatrix} -27/98 & 19/98 & 20/49 \end{bmatrix}^T$

(a)

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 7 \\ 2 & 4 & -3 & -1 \\ -3 & 7 & 2 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & -\frac{2}{3} & \frac{23}{3} & 3 \\ 0 & \frac{26}{3} & -\frac{5}{3} & 1 \\ -3 & 7 & 2 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 0 & \frac{98}{13} & \frac{40}{13} \\ 0 & \frac{26}{3} & -\frac{5}{3} & 1 \\ -3 & 7 & 2 & 3 \end{array}\right]$$

$$\mathbf{r} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}^T$$

(b)

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 2 \\ 2 & 4 & -3 & -1 \\ -3 & 7 & 2 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & -5 & \frac{17}{2} & \frac{5}{2} \\ 2 & 4 & -3 & -1 \\ 0 & 13 & -\frac{5}{2} & \frac{3}{2} \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 0 & 0 & \frac{98}{13} & \frac{40}{13} \\ 2 & 4 & -3 & -1 \\ 0 & 13 & -\frac{5}{2} & \frac{3}{2} \end{array}\right]$$

$$\mathbf{r} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}^T \qquad \mathbf{r} = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}^T$$

$$\mathbf{s} = \begin{bmatrix} 7 & 4 & 7 \end{bmatrix}^T$$

15. exact: $\mathbf{x} = \begin{bmatrix} 1.261157 & 0.0496929 & -0.328744 \end{bmatrix}^T$
    no pivoting: $\mathbf{x} = \begin{bmatrix} 1.26 & 0.0466 & -0.326 \end{bmatrix}^T$
    partial pivoting: $\mathbf{x} = \begin{bmatrix} 1.26 & 0.0471 & -0.327 \end{bmatrix}^T$
    scaled partial pivoting: $\mathbf{x} = \begin{bmatrix} 1.26 & 0.0471 & -0.327 \end{bmatrix}^T$

17. exact: $\mathbf{x} = \begin{bmatrix} 1 & 4 & -1 \end{bmatrix}^T$
    no pivoting: $\mathbf{x} = \begin{bmatrix} 1.684 & 5.000 & -1.000 \end{bmatrix}^T$
    partial pivoting: $\mathbf{x} = \begin{bmatrix} 1.000 & 4.001 & -1.000 \end{bmatrix}^T$
    scaled partial pivoting: $\mathbf{x} = \begin{bmatrix} 0.9995 & 3.999 & -1.000 \end{bmatrix}^T$

19. no pivoting: $\mathbf{x} = \begin{bmatrix} -1.00124 & -0.996035 & -1.00008 \end{bmatrix}^T$
    partial pivoting: $\mathbf{x} = \begin{bmatrix} -0.998305 & -1.00539 & -0.999890 \end{bmatrix}^T$
    scaled partial pivoting: $\mathbf{x} = \begin{bmatrix} -0.999143 & -1.00273 & -0.999943 \end{bmatrix}^T$

Section 3.3 (page 180)

3. (b) from 2(a), $\|\mathbf{x}\|_1 = 8 + \sqrt{2}$; from 2(b), $\|\mathbf{x}\|_1 = 10$; from 2(c), $\|\mathbf{x}\|_1 = 13$; from 2(d), $\|\mathbf{x}\|_1 = 12 + 2\sqrt{3}$; from 2(e), $\|\mathbf{x}\|_1 = 1 + e + \pi$

5. (a) $\{2, 3\}$ (b) $\{0, 1, 5\}$ (c) $\{0, 1, 1\}$ (d) $\{1, -2, 4\}$

7. (b) from 6(a), $\|A\|_1 = 11$; from 6(b), $\|A\|_1 = 5$; from 6(c), $\|A\|_1 = 9$; from 6(d), $\|A\|_1 = 7$

Section 3.4 (page 187)

5. (a) between 10 and 11 (b) between 14 and 15 (c) between 29 and 30
   (d) between 10 and 11 (e) between 27 and 28 (f) between 2 and 3
   (g) between 1 and 2

7. (a) $\kappa_\infty(A) = 6002$ (b) $\kappa_\infty(A) = 200$ (c) $\kappa_\infty(A) = 12$ (d) $\kappa_\infty(A) = 748$

9. (a) $\kappa_\infty(A) = 748$
   (b) $\mathbf{x} = \begin{bmatrix} -1.4 & 1.6 & 2 \end{bmatrix}^T$ and $\tilde{\mathbf{x}} = \begin{bmatrix} -1.15 & 0.24 & 3.3 \end{bmatrix}^T$; therefore, $\|\delta\mathbf{x}\|_\infty / \|\mathbf{x}\|_\infty = 0.68$. The theoretical upper bound from equation (5) is 7.48.
   (c) $\mathbf{x} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ and $\tilde{\mathbf{x}} = \begin{bmatrix} 1.25 & -0.36 & 2.23 \end{bmatrix}^T$; therefore, $\|\delta\mathbf{x}\|_\infty / \|\mathbf{x}\|_\infty = 1.36$. The theoretical upper bound from equation (5) is 1.36.

11. (a) $\kappa_\infty(A) = 30$ (b) $\mathbf{x} = \begin{bmatrix} 4 & -2 & 2 \end{bmatrix}^T$
    (c) $\tilde{\mathbf{x}} = \begin{bmatrix} 6.031299 & -3.463224 & 1.693271 \end{bmatrix}^T$; therefore, $\|\delta\mathbf{x}\|_\infty / \|\mathbf{x}\|_\infty \approx 0.5078$. The theoretical upper bound from equation (5) is 1.6733.
    (d) $\tilde{\mathbf{x}} = \begin{bmatrix} 9.691505 & -5.869598 & 1.204918 \end{bmatrix}^T$; therefore, $\|\delta\mathbf{x}\|_\infty / \|\mathbf{x}\|_\infty \approx 1.4228$. The theoretical upper bound from equation (5) is 7.3333.

13. (a) Single precision has between 7 and 8 significant decimal digits, but the solution is correct to only between 2 and 3 significant digits; hence $\kappa_\infty(A) \approx 10^4\text{-}10^6$.
(b) Double precision has between 15 and 16 significant decimal digits, but the solution is correct to only between 1 and 2 significant digits; hence $\kappa_\infty(A) \approx 10^{13}\text{-}10^{15}$.

15. Double precision has between 15 and 16 significant decimal digits, but the solution is correct to no significant digits; hence $\kappa_\infty(A) \approx 10^{15}\text{-}10^{16}$.

Section 3.5 (page 201)

5. Show that the product of each pair is A; $\mathbf{x} = \begin{bmatrix} -1/3 & 7/9 & -43/45 \end{bmatrix}^T$.

7. (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

9. (b) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

11. (a)

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 7 \end{bmatrix}, \quad P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $\mathbf{x}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$, $\mathbf{x}_2 = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T$, $\mathbf{x}_3 = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T$

13. (a)

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.25 & 1 & 0 & 0 \\ 0.5 & 1.2 & 1 & 0 \\ 0.75 & -0.2 & 1.05882 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 4 & 2 & 6 & -1 \\ 0 & 2.5 & -0.5 & -1.75 \\ 0 & 0 & -3.4 & 4.6 \\ 0 & 0 & 0 & 0.529412 \end{bmatrix},$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) $\mathbf{x}_1 = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T$, $\mathbf{x}_2 = \begin{bmatrix} -19 & 7 & 11 & 9 \end{bmatrix}^T$, $\mathbf{x}_3 = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T$

15. (a)

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 \\ -0.666667 & 0.0784314 & 1 & 0 & 0 \\ 3.16667 & 1.39216 & -3.78333 & 1 & 0 \\ 2.16667 & -0.254902 & -2.96667 & -0.00984219 & 1 \end{bmatrix},$$

$$U = \begin{bmatrix} 6 & 17 & 9 & 5 & 7 \\ 0 & -8.5 & -13.5 & -0.5 & -2.5 \\ 0 & 0 & 7.05882 & -15.6275 & -4.13725 \\ 0 & 0 & 0 & 32.7389 & 9.66111 \\ 0 & 0 & 0 & 0 & 0.0173087 \end{bmatrix},$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) $\mathbf{x}_1 = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \end{bmatrix}^T$, $\mathbf{x}_2 = \begin{bmatrix} -1 & 1 & 1 & 1 & -1 \end{bmatrix}^T$, $\mathbf{x}_3 = \begin{bmatrix} 1 & -2 & 3 & -2 & 1 \end{bmatrix}^T$

17.

	(a)	(b)	(c)	(d)	(e)
F_V	2500	150	4850	1708.3	-41.7
F_H	0	1200	-1200	-500	500
F_1	3354.1	2347.9	4360.3	1397.5	838.5
F_2	3952.8	632.5	5692.1	2239.9	395.3
F_3	-5000	-1500	-8500	-2916.7	-416.7
F_4	7115.1	1581.1	11068.1	5138.7	658.8
F_5	-5500	-2000	-8000	-3083.3	-583.3
F_6	7115.1	1581.1	11068.1	4611.7	1185.9
F_7	-5000	-2500	-7500	-2916.7	-416.7
F_8	3952.8	0	6324.6	2767.0	-131.8
F_9	3354.1	1677.1	5031.2	1956.6	279.5
F_R	2500	1250	3750	1458.3	208.3

Section 3.6 (page 209)

1.

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 6 & -1 & 0 \\ 4 & -11 & 45 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 3.5 & 2.5 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{aligned} \mathbf{x}_1 &= \begin{bmatrix} -\frac{1}{3} & \frac{7}{9} & -\frac{43}{45} \end{bmatrix}^T \\ \mathbf{x}_2 &= \begin{bmatrix} -1 & -1 & 1 \end{bmatrix}^T \\ \mathbf{x}_3 &= \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}^T \end{aligned}$$

3.

$$L = \begin{bmatrix} -3 & 0 & 0 \\ 6 & 12 & 0 \\ 4 & 14/3 & 43/6 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -2/3 & -1/3 \\ 0 & 1 & 1/4 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{aligned} \mathbf{x}_1 &= \begin{bmatrix} 0 & 1 & -5 \end{bmatrix}^T \\ \mathbf{x}_2 &= \begin{bmatrix} 3 & -2 & -1 \end{bmatrix}^T \\ \mathbf{x}_3 &= \begin{bmatrix} -4 & 1 & -3 \end{bmatrix}^T \end{aligned}$$

5.

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 1 & -3 & 3 & 0 \\ -1 & 3 & -3 & 7 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5/3 & 7/3 \\ 0 & 0 & 1 & 5/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \begin{array}{l} \mathbf{x}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \\ \mathbf{x}_2 = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T \\ \mathbf{x}_3 = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T \end{array}$$

9.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 11 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{bmatrix}, \quad \begin{array}{l} \mathbf{x}_1 = \begin{bmatrix} -1/3 & 7/9 & -43/45 \end{bmatrix}^T \\ \mathbf{x}_2 = \begin{bmatrix} -1 & -1 & 1 \end{bmatrix}^T \\ \mathbf{x}_3 = \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}^T \end{array}$$

11.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -4/3 & 7/18 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} -3 & 2 & 1 \\ 0 & 12 & 3 \\ 0 & 0 & 43/6 \end{bmatrix}, \quad \begin{array}{l} \mathbf{x}_1 = \begin{bmatrix} 0 & 1 & -5 \end{bmatrix}^T \\ \mathbf{x}_2 = \begin{bmatrix} 3 & -2 & -1 \end{bmatrix}^T \\ \mathbf{x}_3 = \begin{bmatrix} -4 & 1 & -3 \end{bmatrix}^T \end{array}$$

13.

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 5 & 7 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 7 \end{bmatrix}, \quad \begin{array}{l} \mathbf{x}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \\ \mathbf{x}_2 = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T \\ \mathbf{x}_3 = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T \end{array}$$

15. (a) One approach is to first calculate a Doolittle decomposition and then factor the diagonal elements from the upper triangular matrix.
(b) First, solve $L\mathbf{z} = \mathbf{b}$; then, solve $D\mathbf{y} = \mathbf{z}$; finally, solve $U\mathbf{x} = \mathbf{y}$.
(c) The LDU decomposition requires $\frac{2}{3}n^3 - \frac{2}{3}n$ arithmetic operations; this is $\frac{1}{2}n^2 - \frac{1}{2}n$ more operations than an LU decomposition.
(d) The solve step requires $2n^2 - n$ arithmetic operations, the same as for an LU decomposition.
(e) Solving a system based on an LDU decomposition requires $\frac{1}{2}n^2 - \frac{1}{2}n$ more operations than solving a system with an LU decomposition.

17.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x}_1 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

19.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x}_1 = \begin{bmatrix} -4 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} -25 \\ 10 \\ -5 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$$

21.

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ 4 & 5 & 1.7 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & -0.9 \end{bmatrix},$$

$$U = \begin{bmatrix} 1 & 3 & 1 & -2 \\ 0 & 1 & 1.5 & -3 \\ 0 & 0 & 1 & -1.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} -19 \\ 7 \\ 11 \\ 9 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

Section 3.7 (page 220)

1. (a) both (b) neither (c) neither (d) symmetric positive definite (e) strictly diagonally dominant (f) strictly diagonally dominant

3. (a) $a > 5/19$ (b) $|a| > 1$

5. (a) $b > \frac{1}{3} + \frac{1}{4}a^2$ (b) $|a| < 4$ and $|b| > 1 + |a|$

13. (a) $L = \begin{bmatrix} 4 & 0 & 0 \\ -7 & 2 & 0 \\ 0 & 5 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 7.9375 \\ 4.25 \\ -0.5 \end{bmatrix}$

(b) $L = \begin{bmatrix} 3/2 & 0 & 0 \\ 2 & 3/2 & 0 \\ 1 & 1 & 3/2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 4 \\ -4 \\ 4 \end{bmatrix}$

(c) $L = \begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 0 & -1 & 1 & 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

(d) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 4 & 0 & 0 \\ 3 & 1 & 1 & 0 \\ -2 & 1 & 0 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -9.0625 \\ 0.09375 \\ 6.5 \\ -2.375 \end{bmatrix}$

15. $\mathbf{x} = \begin{bmatrix} 0.043454 & 0.025428 & 0.015127 & 0.009241 & 0.005877 & 0.003955 \end{bmatrix}^T$.

17. (a) The following algorithm requires $\frac{1}{3}n^3 + n^2 - \frac{4}{3}n$ arithmetic operations.

GIVEN: the integer n
the elements in the matrix A, a_{ij}
STEP 1: set $d_{1,1} = a_{1,1}$
STEP 2: for i from 2 to n
set $l_{i,1} = a_{i,1}/d_{1,1}$
end
STEP 3: for i from 2 to $n-1$
STEP 4: for j from 1 to $i-1$
set $v_j = l_{i,j}d_{j,j}$
end
STEP 5: set $d_{i,i} = a_{i,i} - \sum_{j=1}^{i-1} l_{i,j}v_j$
STEP 6: for k from $i+1$ to n
set $l_{i,k} = (a_{i,k} - \sum_{j=1}^{i-1} l_{i,j}v_j)/d_{k,k}$
end
end
STEP 7: for j from 1 to $n-1$
set $v_j = l_{n,j}d_{j,j}$
end
STEP 8: set $d_{n,n} = a_{n,n} - \sum_{j=1}^{n-1} l_{n,j}v_j$
OUTPUT: the elements in the matrices D and L

(b) First, solve $L\mathbf{z} = \mathbf{b}$; then, solve $D\mathbf{y} = \mathbf{z}$; finally, solve $L^T\mathbf{x} = \mathbf{y}$. This procedure requires $2n^2 - n$ arithmetic operations.

19. (a) GIVEN: the integer n
the elements in the matrix A, a_{ij}
STEP 1: set $l_{1,1} = a_{1,1}, l_{2,1} = a_{2,1}, l_{3,1} = a_{3,1}$
STEP 2: set $u_{1,2} = a_{1,2}/l_{1,1}, u_{1,3} = a_{1,3}/l_{1,1}$
STEP 3: set $l_{2,2} = a_{2,2} - l_{2,1}u_{1,2}, l_{3,2} = a_{3,2} - l_{3,1}u_{1,2}, l_{4,2} = a_{4,2}$
STEP 4: set $u_{2,3} = (a_{2,3} - l_{2,1}u_{1,3})/l_{2,2}, u_{2,4} = a_{2,4}/l_{2,2}$
STEP 5: for i from 3 to $n-2$
STEP 6: set $l_{i,i} = a_{i,i} - l_{i,i-2}u_{i-2,i} - l_{i,i-1}u_{i-1,i}$
set $l_{i+1,i} = a_{i+1,i} - l_{i+1,i-1}u_{i-1,i}, l_{i+2,i} = a_{i+2,i}$
STEP 7: set $u_{i,i+1} = (a_{i,i+1} - l_{i,i-1}u_{i-1,i+1})/l_{i,i}, u_{i,i+2} = a_{i,i+2}/l_{i,i}$
end
STEP 8: set $l_{n-1,n-1} = a_{n-1,n-1} - l_{n-1,n-3}u_{n-3,n-1} - l_{n-1,n-2}u_{n-2,n-1}$
set $l_{n,n-1} = a_{n,n-1} - l_{n,n-2}u_{n-2,n-1}$
STEP 9: set $u_{n-1,n} = (a_{n-1,n} - l_{n-1,n-2}u_{n-2,n})/l_{n-1,n-1}$
STEP 10: set $l_{n,n} = a_{n,n} - l_{n,n-2}u_{n-2,n} - l_{n,n-1}u_{n-1,n}$
OUTPUT: the elements in the matrices L and U

(b) The above algorithm requires $10n - 17$ arithmetic operations.
(c) Forward and backward substitution require a total of $9n - 12$ arithmetic operations.

Section 3.8 (page 234)

1. (a) $T_{\text{jac}} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & 0 \end{bmatrix}, \quad T_{\text{gs}} = \begin{bmatrix} 0 & \frac{1}{2} \\ 0 & \frac{1}{6} \end{bmatrix}$
(b) $\rho(T_{\text{jac}}) = \sqrt{6}/6$ and $\rho(T_{\text{gs}}) = 1/6$
(c) Both methods will converge for any choice of $\mathbf{x}^{(0)}$ because the iteration matrix for each method has a spectral radius less than one.

3. (a) $T_{\text{jac}} = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}, \quad T_{\text{gs}} = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & \frac{1}{12} & \frac{1}{6} \\ 0 & \frac{1}{36} & \frac{1}{18} \end{bmatrix}$
(b) $\rho(T_{\text{jac}}) \approx 0.4536$ and $\rho(T_{\text{gs}}) = 5/36$
(c) Both methods will converge for any choice of $\mathbf{x}^{(0)}$ because the iteration matrix for each method has a spectral radius less than one.

5. (a)

$$\begin{aligned} x_1^{(k+1)} &= -\frac{1}{2} + \frac{1}{2}x_2^{(k)} \\ x_2^{(k+1)} &= \frac{3}{4} + \frac{1}{4}x_1^{(k)} - \frac{1}{2}x_3^{(k)} \\ x_3^{(k+1)} &= \frac{5}{6} - \frac{1}{3}x_2^{(k)} \end{aligned} \qquad \mathbf{x}^{(1)} = \begin{bmatrix} -0.500000 \\ 0.750000 \\ 0.833333 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} -0.125000 \\ 0.208333 \\ 0.583333 \end{bmatrix}$$

(b)

$$\begin{aligned} x_1^{(k+1)} &= \frac{4}{3} + \frac{1}{3}x_2^{(k)} - \frac{1}{3}x_3^{(k)} \\ x_2^{(k+1)} &= \frac{13}{6} + \frac{1}{3}x_1^{(k)} + \frac{1}{2}x_3^{(k)} \\ x_3^{(k+1)} &= -\frac{7}{20} - \frac{9}{20}x_1^{(k)} + \frac{7}{20}x_2^{(k)} \end{aligned} \qquad \mathbf{x}^{(1)} = \begin{bmatrix} 1.333333 \\ 2.166667 \\ -0.350000 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} 2.172222 \\ 2.436111 \\ -0.191667 \end{bmatrix}$$

(c)

$$\begin{aligned} x_1^{(k+1)} &= \frac{1}{4} - \frac{1}{2}x_2^{(k)} + \frac{1}{4}x_3^{(k)} \\ x_2^{(k+1)} &= -\frac{1}{4} - \frac{1}{2}x_1^{(k)} - \frac{1}{4}x_3^{(k)} \\ x_3^{(k+1)} &= \frac{1}{4} + \frac{1}{4}x_1^{(k)} + \frac{1}{4}x_2^{(k)} \end{aligned} \qquad \mathbf{x}^{(1)} = \begin{bmatrix} 0.250000 \\ -0.250000 \\ 0.250000 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} 0.437500 \\ -0.437500 \\ 0.375000 \end{bmatrix}$$

(d)

$$\begin{aligned} x_1^{(k+1)} &= \frac{1}{4}x_2^{(k)} \\ x_2^{(k+1)} &= \frac{1}{2} - \frac{1}{2}x_1^{(k)} + \frac{1}{4}x_3^{(k)} \\ x_3^{(k+1)} &= -\frac{3}{4} + \frac{1}{2}x_2^{(k)} + \frac{1}{4}x_4^{(k)} \\ x_4^{(k+1)} &= \frac{1}{4} + \frac{1}{2}x_3^{(k)} \end{aligned} \qquad \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 0.500000 \\ -0.750000 \\ 0.250000 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} 0.125000 \\ 0.312500 \\ -0.437500 \\ -0.125000 \end{bmatrix}$$

7. Jacobi: $\mathbf{x}^{(26)} = \begin{bmatrix} -2.000001 & 2.999999 & -1.000001 & 0.999998 \end{bmatrix}^T$.
   Gauss-Seidel: $\mathbf{x}^{(10)} = \begin{bmatrix} -2.000000 & 3.000000 & -1.000000 & 1.000000 \end{bmatrix}^T$.

9. Jacobi: $\mathbf{x}^{(22)} = \begin{bmatrix} 0.237993 & -0.778016 & 1.716131 & 1.370378 & 1.246918 \end{bmatrix}^T$.
   Gauss-Seidel:
   $\mathbf{x}^{(12)} = \begin{bmatrix} 0.237994 & -0.778016 & 1.716131 & 1.370380 & 1.246920 \end{bmatrix}^T$.

11. Exercise 8: $\mathbf{x}^{(7)} = \begin{bmatrix} 1.000000 & 2.000000 & 3.000000 \end{bmatrix}^T$.
    Exercise 9:
    $\mathbf{x}^{(9)} = \begin{bmatrix} 0.237993 & -0.778017 & 1.716133 & 1.370382 & 1.246921 \end{bmatrix}^T$.

13. (a) $T_{\text{jac}} = \begin{bmatrix} 0 & -2 & 2 \\ -1 & 0 & -1 \\ -2 & -2 & 0 \end{bmatrix}$ and $\rho(T_{\text{jac}}) = 0$
    Since the spectral radius of the iteration matrix is less than one, the method will converge for any choice of $\mathbf{x}^{(0)}$.
    (b) $T_{\text{gs}} = \begin{bmatrix} 0 & -2 & 2 \\ 0 & 2 & -3 \\ 0 & 0 & 2 \end{bmatrix}$ and $\rho(T_{\text{gs}}) = 2$
    Since the spectral radius of the iteration matrix is greater than one, the method will not converge for any choice of $\mathbf{x}^{(0)}$.

17. All calculations were performed with a convergence tolerance of 5×10^{-6}.
    (a) $\mathbf{x}^{(89)} = \begin{bmatrix} 0.131033 & 0.199881 & 0.258373 & 0.308076 & 0.350302 & 0.386183 \end{bmatrix}^T$
    (b) $\mathbf{x}^{(48)} = \begin{bmatrix} 0.131036 & 0.199886 & 0.258383 & 0.308084 & 0.350311 & 0.386187 \end{bmatrix}^T$
    (c)

ω	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
# of iterations	39	32	25	15	20	27	35	56	113

Section 3.9 (page 247)

1.

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{x}^{(1)} = \begin{bmatrix} -\frac{122}{97} \\ \frac{61}{97} \\ -\frac{244}{291} \end{bmatrix}, \quad \mathbf{x}^{(2)} = \begin{bmatrix} -\frac{10345}{8251} \\ \frac{6751}{8251} \\ -\frac{6145}{8251} \end{bmatrix}, \quad \mathbf{x}^{(3)} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

3.

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{x}^{(1)} = \begin{bmatrix} \frac{3113}{2769} \\ -\frac{849}{923} \\ \frac{849}{923} \end{bmatrix}, \quad \mathbf{x}^{(2)} = \begin{bmatrix} \frac{29213}{28243} \\ -\frac{28399}{28243} \\ \frac{26853}{28243} \end{bmatrix}, \quad \mathbf{x}^{(3)} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

5. Jacobi: $\mathbf{x}^{(90)} = \begin{bmatrix} 0.999999 & -0.999998 & 1.999997 & -3.000004 \end{bmatrix}^T$
   Gauss-Seidel: $\mathbf{x}^{(62)} = \begin{bmatrix} 1.000000 & -1.000001 & 2.000001 & -2.999998 \end{bmatrix}^T$

Conjugate gradient:
$\mathbf{x}^{(4)} = \begin{bmatrix} 1.000000 & -1.000000 & 2.000000 & -3.000000 \end{bmatrix}^T$

7. Jacobi:
$\mathbf{x}^{(26)} = \begin{bmatrix} 0.245382 & -0.760776 & 1.583128 & 0.988608 & -0.325739 \end{bmatrix}^T$
Gauss-Seidel:
$\mathbf{x}^{(13)} = \begin{bmatrix} 0.245382 & -0.760776 & 1.583128 & 0.988608 & -0.325739 \end{bmatrix}^T$
SOR ($\omega = 1.0923$):
$\mathbf{x}^{(9)} = \begin{bmatrix} 0.245382 & -0.760776 & 1.583128 & 0.988608 & -0.325739 \end{bmatrix}^T$
Conjugate gradient:
$\mathbf{x}^{(5)} = \begin{bmatrix} 0.245382 & -0.760776 & 1.583128 & 0.988608 & -0.325739 \end{bmatrix}^T$

9. Jacobi: $\mathbf{x}^{(41)} = \begin{bmatrix} 2.999999 & -1.499999 & 2.000001 & 4.000000 \end{bmatrix}^T$
Gauss-Seidel: $\mathbf{x}^{(20)} = \begin{bmatrix} 3.000000 & -1.500000 & 2.000000 & 4.000000 \end{bmatrix}^T$
Conjugate gradient: $\mathbf{x}^{(4)} = \begin{bmatrix} 3.000000 & -1.500000 & 2.000000 & 4.000000 \end{bmatrix}^T$

13. $a_0 = 0.981440, a_1 = 2.860714, a_2 = -2.317099, a_3 = 0.474459$

Section 3.10 (page 258)

1. (a) $\mathbf{F}(\mathbf{x}) = \begin{bmatrix} x_1 - x_2 - x_1^3 \\ x_1 + x_2 - x_2^3 \end{bmatrix}, \quad J(\mathbf{x}) = \begin{bmatrix} 1 - 3x_1^2 & -1 \\ 1 & 1 - 3x_2^2 \end{bmatrix}$
(b) $\mathbf{F}(\mathbf{x}) = \begin{bmatrix} 1 + x_2 - e^{-x_1} \\ x_1^3 - x_2 \end{bmatrix}, \quad J(\mathbf{x}) = \begin{bmatrix} e^{-x_1} & 1 \\ 3x_1^2 & -1 \end{bmatrix}$
(c) $\mathbf{F}(\mathbf{x}) = \begin{bmatrix} 2x_1 - 3x_2 + x_3 - 4 \\ 2x_1 + x_2 - x_3 + 4 \\ x_1^2 + x_2^2 + x_3^2 - 4 \end{bmatrix}, \quad J(\mathbf{x}) = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 1 & -1 \\ 2x_1 & 2x_2 & 2x_3 \end{bmatrix}$

3. (a) $\mathbf{x}^{(1)} = \begin{bmatrix} -0.294118 & 0.176471 \end{bmatrix}^T, \ \mathbf{x}^{(2)} = \begin{bmatrix} -0.0872743 & -0.189131 \end{bmatrix}^T$
(b) $\mathbf{x}^{(1)} = \begin{bmatrix} 0.515386 & -0.453841 \end{bmatrix}^T, \ \mathbf{x}^{(2)} = \begin{bmatrix} 0.360587 & -0.347514 \end{bmatrix}^T$
(c) $\mathbf{x}^{(1)} = \begin{bmatrix} -0.875000 & -1.750000 & 0.500000 \end{bmatrix}^T$
$\mathbf{x}^{(2)} = \begin{bmatrix} -0.865968 & -1.731936 & 0.536128 \end{bmatrix}^T$

5. Newton's method: $\mathbf{x}^{(7)} = \begin{bmatrix} 0.435147 & 0.892027 & 0.246266 \end{bmatrix}^T$
Broyden's method: $\mathbf{x}^{(8)} = \begin{bmatrix} 0.435147 & 0.892027 & 0.246266 \end{bmatrix}^T$

7. Newton's method: $\mathbf{x}^{(4)} = \begin{bmatrix} 0.502379 & 0.150579 \end{bmatrix}^T$
Broyden's method: $\mathbf{x}^{(5)} = \begin{bmatrix} 0.502379 & 0.150579 \end{bmatrix}^T$

9. Newton's method: $\mathbf{x}^{(4)} = \begin{bmatrix} 0.486405 & 0.233726 \end{bmatrix}^T$
Broyden's method: $\mathbf{x}^{(5)} = \begin{bmatrix} 0.486405 & 0.233726 \end{bmatrix}^T$

11. (a) $\begin{bmatrix} -0.500000 & 0.606531 \end{bmatrix}^T, \quad \begin{bmatrix} 0.451907 & 1.571306 \end{bmatrix}^T$
(b) $\begin{bmatrix} \pm 0.816497 & \pm 0.408248 & \mp 0.408248 & 3.000000 \end{bmatrix}^T,$

$\begin{bmatrix} 0 & \pm 0.707107 & \pm 0.707106 & 1.000000 \end{bmatrix}^T$

(c) $\begin{bmatrix} 1.709427 & -1.441478 \end{bmatrix}^T$, $\begin{bmatrix} -1.441478 & 1.709427 \end{bmatrix}^T$

13. (a) $\mathbf{x}^{(4)} = \begin{bmatrix} 0.10987 & 0.49001 \end{bmatrix}^T$

    (b) $\mathbf{x}^{(5)} = \begin{bmatrix} 0.2388 & 0.0869 & 0.0330 & 0.0539 & 0.0869 & 0.1519 & 0.2388 \end{bmatrix}^T$

15. 0.29979 moles (0.14962 moles from the first reaction, 0.15017 moles from the second)