Answers to Selected Exercises for Chapter 6

Section 6.1 (page 437)

1. To two decimal places $\mu = 0.29$, with the exception of small regions around $\theta = \frac{\pi}{4}$, $\theta = 4\pi$ and $\theta = 5\pi$, where $\mu = 0.28$ to two decimal places.
3. Using a not-a-knot cubic spline, the maximum sound speed of water is $a = 1554.07$ m/s at a temperature of 78.536°C.
5. Using a not-a-knot cubic spline, minimum thermal resistance is achieved with an insulation thickness of 5.879 mm.
7. Using either a not-a-knot cubic spline or the 4th degree interpolating polynomial

   (a) $\dfrac{\partial\theta}{\partial u} = 11.33^\circ/\mu\text{m}$ (b) $\dfrac{\partial\theta}{\partial u} = 44.79$

Section 6.2 (page 445)

5. (b) $-hf'''(\xi)$, where $x_0 < \xi < x_0 + 2h$.

   (c)

h	$\frac{f(x_0)-2f(x_0+h)+f(x_0+2h)}{h^2}$	error
1	2.952492	1.952492
0.1	1.106092	0.106092
0.01	1.010060	0.010060
0.001	1.001000	0.001000

7. (a)

   $$f'(x_0) \approx \frac{f(x_0-2h) - 6f(x_0-h) + 3f(x_0) + 2f(x_0+h)}{6h}$$

   (b) The error term is $-\frac{h^3}{12}f^{(4)}(\xi)$, where $x_0 - 2h < \xi < x_0 + h$; consequently, provided f has four continuous derivatives near x_0, the finite difference formula has rate of convergence $O(h^3)$.
9. (a)

   $$f''(x_0) = \frac{2}{h^2}\left(\frac{f(x_0-\alpha h)}{\alpha(\alpha+1)} - \frac{f(x_0)}{\alpha^2} + \frac{f(x_0+h)}{\alpha+1}\right)$$

   (b) For $\alpha \neq 1$, the error term is $\frac{h}{3}(\alpha-1)f'''(\xi)$; whereas, for $\alpha = 1$, the error term is $-\frac{h^2}{12}f^{(4)}(\xi)$.

11.

$f(x)$	Formula (a)	Formula (b)	Formula (c)
1	0	0	0
x	1	1	1
x^2	$2x_0$	$2x_0$	$2x_0$
x^3	$3x_0^2 - 2h^2$	$3x_0^2 - 2h^2$	$3x_0^2 + h^2$

13. (a) With $h = 1, f'(1) \approx 8$; with $h = 0.1, f'(1) \approx 4.31$; with $h = 0.01, f'(1) \approx 4.0301$; with $h = 0.001, f'(1) \approx 4.003001$. This data suggests first-order convergence.
(b) With $h = 1, f'(0) \approx 2$; with $h = 0.1, f'(0) \approx 1.01$; with $h = 0.01, f'(0) \approx 1.0001$; with $h = 0.001, f'(0) \approx 1.000001$. This data suggests second-order convergence.
(c) The rate of convergence in part (a) is what one would expect from the given formula; the rate of convergence is higher than expected in part (b) because $f''(0) = 0$.

15. (a) $\frac{20\epsilon}{3h} + \frac{h^3M}{4}$ (b) $h = \sqrt[4]{80\epsilon/9M} = 0.020499$ using $M = 3$

Section 6.3 (page 453)

1. $h \approx 0.00034$

3. Second column, 0.7117737509 and 0.7076386896; third column, 0.7070479668

5. First column, 0.6941218505; second column, 0.6944444443 and 0.6932539683; third column, 0.6931479014

7. First column, 0.3678794407 and 0.8824969025; second column, 0.9861930220; third column, 0.9863672489

11. (b) $f''(0) \approx 2.000000682$, error $= 6.82 \times 10^{-7}$

13. (a) With $h = 1/4, f'(\pi) \approx -0.9896158370$; with $h = 1/8$,
$f'(\pi) \approx -0.9973978671$. The extrapolated value is $f'(\pi) \approx -1.0051798971$.
(b) The error in the approximation associated with $h = 1/4$ is 0.0103841630, the error in the approximation associated with $h = 1/8$ is 0.0026021329, and the error in the extrapolated value is 0.0051798971. The extrapolated value is not a better approximation to $f'(\pi)$ because the original approximations are second-order accurate ($0.0103841630/0.0026021329 \approx 4$), which is better than expected. This better than expected performance for the original approximations arises because $f''(\pi) = 0$.

Section 6.4 (page 465)

1.

	Trapezoidal Rule	Error	Error Bound
$\int_1^2 \frac{1}{x} dx$	0.750000	0.056853	0.166667
$\int_0^1 e^{-x} dx$	0.683940	0.051819	0.083333
$\int_0^1 \frac{1}{1+x^2} dx$	0.750000	0.035398	0.166667
$\int_0^1 \tan^{-1} x dx$	0.392699	0.046125	0.054127

3.

	Midpoint Rule	Error	Error Bound
$\int_1^2 \frac{1}{x}dx$	0.666667	0.026481	0.083333
$\int_0^1 e^{-x}dx$	0.606531	0.025590	0.041667
$\int_0^1 \frac{1}{1+x^2}dx$	0.800000	0.014602	0.083333
$\int_0^1 \tan^{-1} x dx$	0.463648	0.024823	0.027063

5.

$f(x)$	$I(f)$	$I_{1,\text{open}}(f)$
1	$b-a$	$b-a$
x	$\frac{1}{2}(b^2-a^2)$	$\frac{1}{2}(b^2-a^2)$
x^2	$\frac{1}{3}(b^3-a^3)$	$\frac{5}{18}(b^3+\frac{3}{5}b^2a-\frac{3}{5}ba^2-a^3)$

7. (a) $A_0 = \frac{3}{2}, A_1 = 0, A_2 = \frac{1}{2}$ (b) degree of precision equals 2

9. degree of precision equals 3

13. (a)

$f(x)$	$I(f)$	$I_{4,\text{closed}}(f)$
1	$b-a$	$b-a$
x	$\frac{1}{2}(b^2-a^2)$	$\frac{1}{2}(b^2-a^2)$
x^2	$\frac{1}{3}(b^3-a^3)$	$\frac{1}{3}(b^3-a^3)$
x^3	$\frac{1}{4}(b^4-a^4)$	$\frac{1}{4}(b^4-a^4)$
x^4	$\frac{1}{5}(b^5-a^5)$	$\frac{1}{5}(b^5-a^5)$
x^5	$\frac{1}{6}(b^6-a^6)$	$\frac{1}{6}(b^6-a^6)$
x^6	$\frac{1}{7}(b^7-a^7)$	$\frac{55}{384}(b^7-\frac{1}{55}b^6a+\frac{3}{55}b^5a^2-\frac{1}{11}b^4a^3$ $+\frac{1}{11}b^3a^4-\frac{3}{55}b^2a^5+\frac{1}{55}ba^6-a^7)$

(b) $-\frac{1}{1935360}(b-a)^7 f^{(6)}(\xi)$

15. (b)

$f(x)$	$I(f)$	$I_{4,\text{closed}}(f)$
1	$b-a$	$b-a$
x	$\frac{1}{2}(b^2-a^2)$	$\frac{1}{2}(b^2-a^2)$
x^2	$\frac{1}{3}(b^3-a^3)$	$\frac{1}{3}(b^3-a^3)$
x^3	$\frac{1}{4}(b^4-a^4)$	$\frac{1}{4}(b^4-a^4)$
x^4	$\frac{1}{5}(b^5-a^5)$	$\frac{731}{3750}(b^5+\frac{95}{731}b^4a-\frac{190}{731}b^3a^2$ $+\frac{190}{731}b^2a^3-\frac{95}{731}ba^4-a^5)$

(c) $\frac{19}{90000}(b-a)^5 f^{(4)}(\xi)$

Section 6.5 (page 479)

1. Use the Extreme Value Theorem and the Intermediate Value Theorem to show there exists a ξ between a and b such that $\frac{1}{m}\sum_{j=1}^{m} f^{(4)}(\xi_j) = f^{(4)}(\xi)$.
3. (b) $2^4 = 16$
5. Let M_h denote the composite midpoint rule approximation computed with a subinterval size of h, and let e_h denote the corresponding approximation error.

(a) $\int_0^1 \sqrt{1+x^3}\,dx$			(b) $\int_0^\pi \sin x\,dx$		
h	M_h	$\frac{M_h - M_{h/2}}{M_{h/2} - M_{h/4}}$	h	M_h	$\lvert e_{2h}/e_h \rvert$
1	1.060660	4.606	π	3.141593	
$\frac{1}{2}$	1.100103	4.100	$\frac{\pi}{2}$	2.221441	5.155
$\frac{1}{4}$	1.108667	4.025	$\frac{\pi}{4}$	2.052344	4.230
$\frac{1}{8}$	1.110756	4.006	$\frac{\pi}{8}$	2.012909	4.055
$\frac{1}{16}$	1.111275	4.002	$\frac{\pi}{16}$	2.003216	4.014
$\frac{1}{32}$	1.111405		$\frac{\pi}{32}$	2.000803	4.003
$\frac{1}{64}$	1.111437		$\frac{\pi}{64}$	2.000201	4.001

7.

h	T_h	$\lvert e_{2h}/e_h \rvert$	M_h	$\lvert e_{2h}/e_h \rvert$	S_h	$\lvert e_{2h}/e_h \rvert$
$\frac{1}{2}$	0.645235		0.625584		0.63233368	
$\frac{1}{4}$	0.635409	3.988	0.630477	3.978	0.63213418	15.652
$\frac{1}{8}$	0.632943	3.997	0.631709	3.995	0.63212141	15.911
$\frac{1}{16}$	0.632326	3.999	0.632018	3.999	0.63212061	15.978
$\frac{1}{32}$	0.632172	4.000	0.632095	4.000	0.63212056	15.994
$\frac{1}{64}$	0.632133	4.000	0.632114	4.000	0.63212056	15.999

9.

h	T_h	$\frac{T_h - T_{h/2}}{T_{h/2} - T_{h/4}}$	M_h	$\frac{M_h - M_{h/2}}{M_{h/2} - M_{h/4}}$	S_h	$\frac{S_h - S_{h/2}}{S_{h/2} - S_{h/4}}$
$\frac{1}{2}$	0.656528	4.007	0.660733	4.012	0.65935105	16.222
$\frac{1}{4}$	0.658630	4.002	0.659680	4.003	0.65933121	16.055
$\frac{1}{8}$	0.659155	4.000	0.659417	4.001	0.65932999	16.014
$\frac{1}{16}$	0.659286	4.000	0.659352	4.000	0.65932991	16.003
$\frac{1}{32}$	0.659319		0.659335		0.65932991	
$\frac{1}{64}$	0.659327		0.659331		0.65932991	

11.

h	T_h	$\lvert e_{2h}/e_h \rvert$	M_h	$\lvert e_{2h}/e_h \rvert$	S_h	$\lvert e_{2h}/e_h \rvert$
2	34.422205		31.780399		32.56294014	
1	33.101302	4.039	32.448858	4.069	32.66100133	18.309
$\frac{1}{2}$	32.775080	4.009	32.612430	4.016	32.66633974	17.329
$\frac{1}{4}$	32.693755	4.002	32.653121	4.004	32.66664655	16.254
$\frac{1}{8}$	32.673438	4.001	32.663281	4.001	32.66666541	16.060
$\frac{1}{16}$	32.668359	4.000	32.665820	4.000	32.66666659	16.015

13. (a) $n = 82$ (b) $n = 117$

15. 0.63215 using trapezoidal rule with $n = 41$; 0.63209 using midpoint rule with $n = 29$; 0.63213 using Simpson’s rule with $n = 4$

17. 0.65930 using trapezoidal rule with $n = 20$; 0.65935 using midpoint rule with $n = 15$; 0.65935 using Simpson’s rule with $n = 2$

19. 32.66670 using trapezoidal rule with $n = 449$; 32.66663 using midpoint rule with $n = 318$; 32.66665 using Simpson’s rule with $n = 16$

23. (a) rate of convergence is roughly $O(h^{1.5})$
(b) rate of convergence is $O(h^4)$
(c) rate of convergence is $O(h^2)$
(d) The rate of convergence is lower than expected in part (a) because the derivatives of $f(x) = \sin(\sqrt{\pi x})$ are not bounded at $x = 0$. The rate of convergence is better than expected in part (b) because $f'(\pi/4) = f'(9\pi/4)$.

25. 0.70135 (using Simpson's rule with $n = 2$) and 0.52634 (using Simpson's rule with $n = 4$)

27. (a) 686.105 (b) 683.784 using a not-a-knot cubic spline

29. 0.90868 using the trapezoidal rule, 0.90701 using Simpson's rule

Section 6.6 (page 492)

1.

	2-Point Gaussian	Error	Error Bound
$\int_{-1}^{1} e^{-x}\,dx$	2.342696	0.007706	0.020135
$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$	1.500000	0.070796	0.177778
$\int_{0}^{\pi} \sin x\,dx$	1.935820	0.064180	0.070838
$\int_{0}^{1} \tan^{-1} x\,dx$	0.438029	0.000796	0.001081

3.

	2-Point Gaussian	Error	Error Bound
$\int_{-1}^{1} e^{-x}\,dx$	2.349875	0.000527	0.001258
$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$	1.573770	0.002974	0.011111
$\int_{0}^{\pi} \sin x\,dx$	1.999423	0.000577	0.000875
$\int_{0}^{1} \tan^{-1} x\,dx$	0.438817	0.000007	0.000013

5. (b)
$$\frac{b-a}{2}\Bigg[\frac{5}{9}f\left(\frac{a+b}{2}-\sqrt{\frac{3}{5}}\frac{b-a}{2}\right)+\frac{8}{9}f\left(\frac{a+b}{2}\right)$$
$$+\frac{5}{9}f\left(\frac{a+b}{2}+\sqrt{\frac{3}{5}}\frac{b-a}{2}\right)\Bigg]+\frac{(b-a)^7}{2016000}f^{(6)}(\xi)$$

(c)
$$\frac{h}{2}\sum_{j=1}^{n}\left[\frac{5}{9}f\left(x_j-\frac{h}{2}-\sqrt{\frac{3}{5}}\frac{h}{2}\right)+\frac{8}{9}f\left(x_j-\frac{h}{2}\right)\right.$$
$$\left.+\frac{5}{9}f\left(x_j-\frac{h}{2}+\sqrt{\frac{3}{5}}\frac{h}{2}\right)\right]+\frac{(b-a)h^6}{2016000}f^{(6)}(\xi)$$

7.

	3-Point Gaussian	Error	Error Bound
$\int_{-1}^{1} e^{-x}\,dx$	2.350337	0.000065	0.000173
$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$	1.583333	0.012537	0.045714
$\int_{0}^{\pi} \sin x\,dx$	2.001389	0.001389	0.001498
$\int_{0}^{1} \tan^{-1} x\,dx$	0.438838	0.000014	0.000050

9.

h	$GQ2_h$	$\frac{GQ2_h - GQ2_{h/2}}{GQ2_{h/2} - GQ2_{h/4}}$	$GQ3_h$	$\frac{GQ3_h - GQ3_{h/2}}{GQ3_{h/2} - GQ3_{h/4}}$
1	1.112797	24.379	1.111406	1821.514
$\frac{1}{2}$	1.111504	16.085	1.111448	15.538
$\frac{1}{4}$	1.111451	16.080	1.111448	63.616
$\frac{1}{8}$	1.111448	16.020	1.111448	64.011
$\frac{1}{16}$	1.111448	16.005	1.111448	64.880

11.

h	$GQ2_h$	$\lvert e_{2h}/e_h \rvert$	$GQ3_h$	$\lvert e_{2h}/e_h \rvert$
$\frac{1}{2}$	0.693077	11.900	0.693146	37.223
$\frac{1}{4}$	0.693142	14.432	0.693147	52.080
$\frac{1}{8}$	0.693147	15.535	0.693147	60.141
$\frac{1}{16}$	0.693147	15.877	0.693147	62.947
$\frac{1}{32}$	0.693147	15.969	0.693147	63.841

13.

h	$GQ2_h$	$\lvert e_{2h}/e_h \rvert$	$GQ3_h$	$\lvert e_{2h}/e_h \rvert$
$\frac{1}{2}$	0.438784	19.827	0.438825	47.684
$\frac{1}{4}$	0.438822	17.360	0.438825	82.294
$\frac{1}{8}$	0.438824	16.271	0.438825	66.891
$\frac{1}{16}$	0.438825	16.065	0.438825	64.654
$\frac{1}{32}$	0.438825	16.016	0.438825	63.564

15.

h	$GQ2_h$	$\frac{GQ2_h - GQ2_{h/2}}{GQ2_{h/2} - GQ2_{h/4}}$	$GQ3_h$	$\frac{GQ3_h - GQ3_{h/2}}{GQ3_{h/2} - GQ3_{h/4}}$
1	0.924071	36.749	0.927184	80.886
$\frac{1}{2}$	0.926955	21.215	0.927039	266.763
$\frac{1}{4}$	0.927033	16.329	0.927037	57.477
$\frac{1}{8}$	0.927037	16.092	0.927037	63.107
$\frac{1}{16}$	0.927037	16.023	0.927037	63.911

17. 0.693142 using two-point Gaussian quadrature with $n = 4$; 0.693146 using three-point Gaussian quadrature with $n = 2$

19. 0.438817 using two-point Gaussian quadrature with $n = 3$; 0.438838 using three-point Gaussian quadrature with $n = 1$

21. 0.927033 using two-point Gaussian quadrature with $n = 4$; 0.927037 using three-point Gaussian quadrature with $n = 2$

23. 1.111459 using two-point Gaussian quadrature with $n = 3$; 1.111448 using three-point Gaussian quadrature with $n = 2$

25. (a) rate of convergence is roughly $O(h^{1.5})$ (b) rate of convergence is $O(h^4)$
(c) rate of convergence is $O(h^4)$
(d) The rate of convergence is lower than expected in part (a) because the derivatives of $f(x) = \sin(\sqrt{\pi x})$ are not bounded at $x = 0$.

27. (a) rate of convergence is $O(h^4)$ (b) rate of convergence is $O(h^6)$
(c) rate of convergence is $O(h^4)$
(d) The rate of convergence is better than expected in part (b) because $f'''(3-\sqrt{3}) = f'''(3+\sqrt{3})$.

29. (a) $w_1 = w_2 = w_3 = \frac{\pi}{3}$; $x_1 = -\frac{\sqrt{3}}{2}, x_2 = 0, x_3 = \frac{\sqrt{3}}{2}$
(b) 2.404071

Section 6.7 (page 502)

1. trapezoidal rule, 117302 function evaluations; midpoint rule, 82945; Simpson's rule, 325; two-point Gaussian quadrature, 294

3. trapezoidal rule, 251194 function evaluations; midpoint rule, 177622; Simpson's rule, 257; two-point Gaussian quadrature, 232

5. absolute error in final approximation: 2.901×10^{-7}

8.3890560989			
6.9128098779	6.4207278042		
6.5216101095	6.3912101867	6.3891937253	
6.4222978214	6.3891937254	6.3890592946	6.3890563890

7. absolute error in final approximation: 1.826×10^{-5}

1.3333333333			
1.1666666667	1.1111111112		
1.1166666667	1.1000000000	1.0992592593	
1.1032106782	1.0987253487	1.0986403719	1.0986305483

9. (a)

0.1839397206			
0.1677861928	0.1624016835		
0.1624884051	0.1607224759	0.1606105287	
0.1610798961	0.1606103931	0.1606029209	0.1606028001

(b) The error estimate is 9.6607×10^{-7}; the actual error is 5.987×10^{-9}.
(c) $n = 5277$ to guarantee an accuracy of 5.987×10^{-9}

11. (a)

0.3926990817			
0.4281733453	0.4399980999		
0.4362066157	0.4388843724	0.4388101239	
0.4381726803	0.4388280352	0.4388242794	0.4388245041

(b) The error estimate is 1.7975×10^{-6}; the actual error is 6.906×10^{-8}.
(c) $n = 886$ to guarantee an accuracy of 6.906×10^{-8}

13. (a)

0.7853981634			
0.7180216528	0.6955628160		
0.6995166680	0.6933483397	0.6932007079	
0.6947499777	0.6931610810	0.6931485971	0.6931477699

(b) The error estimate is 6.6172×10^{-6}; the actual error is 5.894×10^{-7}.
(c) $n = 741$ to guarantee an accuracy of 5.894×10^{-7}

15. 0.9270373576; 17 function evaluations
17. 0.3102683012; 17 function evaluations
19. 0.1264387818; 33 function evaluations
23. (a) 0.76734 (b) $M = 0.3440$

Section 6.8 (page 517)

1.

	$S(a,b)$	$S(a,c)$	$S(c,b)$	Error Estimate	Error
$\int_0^1 e^{-x}\,dx$	0.632334	0.393478	0.238656	1.995×10^{-5}	1.362×10^{-5}
$\int_1^2 \frac{1}{x}\,dx$	0.694444	0.405556	0.287698	1.190×10^{-4}	1.068×10^{-4}
$\int_0^4 x\sqrt{x^2+9}\,dx$	32.562940	6.620071	26.040930	9.806×10^{-3}	5.665×10^{-3}
$\int_0^1 \tan^{-1} x\,dx$	0.439998	0.120297	0.318588	1.114×10^{-4}	5.980×10^{-5}

3.

	$GQ2(a,b)$	$GQ2(a,c)$	$GQ2(c,b)$	Error Estimate	Error
$\int_0^1 e^{-x}\,dx$	0.631979	0.393464	0.238648	1.327×10^{-5}	9.073×10^{-6}
$\int_1^2 \frac{1}{x}\,dx$	0.692308	0.405405	0.287671	7.689×10^{-5}	7.054×10^{-5}
$\int_0^4 x\sqrt{x^2+9}\,dx$	32.736661	6.626742	26.043727	6.619×10^{-3}	3.802×10^{-3}
$\int_0^1 \tan^{-1} x\,dx$	0.438029	0.120222	0.318563	7.554×10^{-5}	4.012×10^{-5}

5.

	$S(a,b)$	$B(a,b)$	$\lvert S(a,b) - B(a,b)\rvert$	$\lvert I - S(a,b)\rvert$
$\int_0^1 e^{-x}\,dx$	0.632334	0.632121	0.000213	0.000213
$\int_1^2 \frac{1}{x}\,dx$	0.694444	0.693175	0.001270	0.001297
$\int_0^4 x\sqrt{x^2+9}\,dx$	32.562940	32.667539	0.104599	0.103727
$\int_0^1 \tan^{-1} x\,dx$	0.439998	0.438810	0.001188	0.001174

7. composite Simpson’s rule: 169681 function evaluations; composite two-point Gaussian quadrature: 153324 function evaluations

9. using Simpson’s rule as the basic quadrature

		Function Evaluations	
	$\int_0^5 \frac{1}{\sqrt{1+x^3}}\,dx$	Adaptive	Composite
$\epsilon = 5 \times 10^{-7}$	1.9104465	97	151
$\epsilon = 5 \times 10^{-11}$	1.91044647715	1013	1495

11. using Simpson’s rule as the basic quadrature

		Function Evaluations	
	$\int_0^2 e^{-x}\sin(x^2\cos e^{-x})\,dx$	Adaptive	Composite
$\epsilon = 5 \times 10^{-7}$	0.2813862	61	59
$\epsilon = 5 \times 10^{-11}$	0.28138616866	677	579

13. using Simpson’s rule as the basic quadrature

		Function Evaluations	
	$\int_0^1 \frac{u^7}{1+u^{14}}\,du$	Adaptive	Composite
$\epsilon = 5 \times 10^{-7}$	0.0959715	57	93
$\epsilon = 5 \times 10^{-11}$	0.09597143321	537	915

15. using Simpson's rule as the basic quadrature

		Function Evaluations	
	$\int_0^1 \frac{1}{1+e^x}\,dx$	Adaptive	Composite
$\epsilon = 5 \times 10^{-7}$	0.3798854	9	9
$\epsilon = 5 \times 10^{-11}$	0.37988549303	97	63

17. (a) 0.8497261, using 157 function evaluations
    (b) 0.8497262, using 33 function evaluations

19. (a)

x	$c(x)$	$s(x)$
0.0	0.00000	0.00000
0.2	0.19992	0.00419
0.4	0.39748	0.03336
0.6	0.58109	0.11054
0.8	0.72284	0.24934
1.0	0.77989	0.43826
1.2	0.71544	0.62340
1.4	0.54310	0.71352
1.6	0.36546	0.63889
1.8	0.33363	0.45094
2.0	0.48822	0.34342

(b) $c(x) = 0.5$ at $x = 0.5083$ and $x = 1.4432$; $s(x) = 0.5$ at $x = 1.0622$ and $x = 1.7494$

21. 0.575877 and 0.551891

Section 6.9 (page 531)

1. (a) trapezoidal rule, $O(h^{1.438})$; Simpson's rule, $O(h^{1.496})$; midpoint rule, $O(h^{1.382})$; two-point Gaussian, $O(h^{1.496})$
   (b) after making the substitution $x = u^2$: trapezoidal rule, $O(h^2)$; Simpson's rule, $O(h^4)$; midpoint rule, $O(h^2)$; two-point Gaussian, $O(h^4)$

3. (a) trapezoidal rule, $O(h^{1.479})$; Simpson's rule, $O(h^{1.503})$; midpoint rule, $O(h^{1.461})$; two-point Gaussian, $O(h^{1.503})$
   (b) after making the substitution $x = u^2$: trapezoidal rule, $O(h^2)$; Simpson's rule, $O(h^4)$; midpoint rule, $O(h^2)$; two-point Gaussian, $O(h^4)$

5. 0.6718000324; derivatives are discontinuous at $x = 0$; let $x = u^7$

7. 0.9064024771; infinite limit of integration; split integration interval into $[0, 1] \cup [1, \infty)$ and on $[1, \infty)$ make the substitution $x = \frac{1}{u}$

9. 3.1415926536; algebraic discontinuity at $x = 0$ and infinite limit of integration; split integration interval into $[0, 1] \cup [1, \infty)$ and on $[0, 1]$ make the substitution $x = u^2$, while on $[1, \infty)$ make the substitution $x = \frac{1}{u^2}$
11. -0.6973166594; removable discontinuity at $x = 0$
13. 1.8137993642; infinite limits of integration; make the substitution $x = \tan\theta$
15. (a) 3.9774632605 (b) 1.1780972451 (c) -0.9558049902
17. $G(1) \approx 0.3117654831, G(5) \approx 0.01773806110$
19. 2.4041138063 and 6.4939394023