Problem 2.2
(a) "Horizontal" components cancel. Net vertical field is: \( E_z = \frac{1}{4\pi \varepsilon_0} \frac{2qz}{(z^2 + (\frac{d}{2})^2)^{3/2}} \).

Here \( z^2 = z^2 + (\frac{d}{2})^2 \); \( \cos \theta = \frac{z}{L} \), so

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{2qz}{(z^2 + (\frac{d}{2})^2)^{3/2}} \hat{z}.
\]

When \( z \gg d \) you're so far away it just looks like a single charge \( 2q \); the field should reduce to \( E = \frac{1}{4\pi \varepsilon_0} \frac{2qz}{(2d)^{3/2}} \hat{z} \). And it does (just set \( d \to 0 \) in the formula).

(b) This time the "vertical" components cancel, leaving

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{q d}{(z^2 + (\frac{d}{2})^2)^{3/2}} \hat{x},
\]

From far away, \( (z \gg d) \), the field goes like \( E \approx \frac{1}{4\pi \varepsilon_0} \frac{q d}{z^2} \hat{z} \), which, as we shall see, is the field of a dipole. (If we set \( d \to 0 \), we get \( E = 0 \), as is appropriate; to the extent that this configuration looks like a single point charge from far away, the net charge is zero, so \( E \to 0 \).

Problem 2.3

\[
E_z = \frac{1}{4\pi \varepsilon_0} \int_0^L \frac{\lambda dx}{\left( z^2 + x^2 \right)^{3/2}} \cos \theta; \quad \left( z^2 = z^2 + x^2; \quad \cos \theta = \frac{z}{L} \right)
\]

\[
= \frac{1}{4\pi \varepsilon_0} \frac{\lambda z}{L} \left[ \frac{1}{\sqrt{z^2 + L^2}} \right]_0^L = \frac{1}{4\pi \varepsilon_0} \frac{\lambda L}{\sqrt{z^2 + L^2}}
\]

\[
E_x = \frac{1}{4\pi \varepsilon_0} \lambda \int_0^L \frac{d\theta}{(z^2 + \theta^2)^{3/2}}
\]

\[
= \frac{1}{4\pi \varepsilon_0} \lambda \left[ \frac{1}{\sqrt{z^2 + \theta^2}} \right]_0^L = \frac{1}{4\pi \varepsilon_0} \lambda \frac{1}{\sqrt{z^2 + L^2}}.
\]

For \( z \gg L \) you expect it to look like a point charge \( q = \lambda L \); \( E \to \frac{1}{4\pi \varepsilon_0} \frac{\lambda L}{z} \hat{z} \). It checks, for with \( z \gg L \) the \( \hat{\theta} \) term \( \to 0 \), and the \( \hat{z} \) term \( \to \frac{1}{4\pi \varepsilon_0} \frac{\lambda L}{z} \hat{z} \).

Problem 2.5

"Horizontal" components cancel, leaving: \( E = \frac{1}{4\pi \varepsilon_0} \int_0^L \frac{\lambda dx}{\left( z^2 + x^2 \right)^{3/2}} \hat{z} \).

Here, \( z^2 = r^2 + z^2 \), \( \cos \theta = \frac{z}{r} \) (both constants), while \( \int d\ell = 2\pi r \). So

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{\lambda (2\pi r) z}{(r^2 + z^2)^{3/2}} \hat{z}.
\]

Problem 2.6

Break into rings of radius \( r \), and thickness \( dr \), and use Prob. 2.5 to express the field of each ring. Total charge of a ring is \( \sigma \cdot 2\pi r \cdot dr = \lambda \cdot 2\pi r \), so \( \lambda = \sigma dr \) is the "line charge" of each ring.

\[
E_{\text{ring}} = \frac{1}{4\pi \varepsilon_0} \frac{(\sigma dr) 2\pi rz}{(r^2 + z^2)^{3/2}}; \quad E_{\text{disk}} = \frac{1}{4\pi \varepsilon_0} \frac{2\pi \sigma z}{(r^2 + z^2)^{3/2}} \int_0^R \frac{r}{(r^2 + z^2)^{3/2}} dr.
\]

\[
E_{\text{disk}} = \frac{1}{4\pi \varepsilon_0} \frac{2\pi \sigma z}{(r^2 + z^2)^{3/2}} \left[ \frac{1}{z} - \frac{1}{\sqrt{r^2 + z^2}} \right] \hat{z}.
\]

For \( R \gg z \) the second term \( \to 0 \), so \( E_{\text{plane}} = \frac{1}{4\pi \varepsilon_0} \frac{2\pi \sigma z}{(r^2 + z^2)^{3/2}} \hat{z} \).

For \( R \gg R, \frac{1}{\sqrt{R^2 + z^2}} = \frac{1}{z} \left( 1 + \frac{R^2}{z^2} \right)^{-1/2} \approx \frac{1}{z} \left( 1 - \frac{1}{2} \frac{R^2}{z^2} \right), \) so \( [ \approx \frac{1}{z} - \frac{1}{2} \frac{R^2}{z^2} = \frac{R^2}{2z^2}, \)

and \( E = \frac{1}{4\pi \varepsilon_0} \frac{2\pi R^2 \sigma}{2z^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{z^2}, \) where \( Q = \pi R^2 \sigma \).
Problem 2.8

According to Prob. 2.7, all shells interior to the point (i.e. at smaller \( r \)) contribute as though their charge were concentrated at the center, while all exterior shells contribute nothing. Therefore:

\[
E(r) = \frac{1}{4\pi \varepsilon_0} \frac{Q_{\text{int}}}{r^2} \hat{r},
\]

where \( Q_{\text{int}} \) is the total charge interior to the point. Outside the sphere, all the charge is interior, so

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \hat{r}.
\]

Inside the sphere, only that fraction of the total which is interior to the point counts:

\[
Q_{\text{int}} = \frac{4}{3} \pi r^3 Q = \frac{r^3}{R^3} Q, \quad \text{so} \quad E = \frac{1}{4\pi \varepsilon_0} \frac{r^3}{R^3} Q \frac{1}{r^2} \hat{r} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^3} \hat{r}.
\]

Problem 2.9

(a) \( \rho = \varepsilon_0 \nabla \cdot E = \varepsilon_0 \frac{1}{\varepsilon_0} k \frac{d}{dr} \left( r^2 \cdot kr^2 \right) = \varepsilon_0 \frac{1}{\varepsilon_0} k (5r^4) = 5\varepsilon_0 kr^2 \).

Problem 2.10

Think of this cube as one of 8 surrounding the charge. Each of the 24 squares which make up the surface of this larger cube gets the same flux as every other one, so:

\[
\int_{\text{one face}} E \cdot da = \frac{1}{24} \int_{\text{whole large cube}} E \cdot da.
\]

The latter is \( \frac{1}{\varepsilon_0} q \), by Gauss's law. Therefore

\[
\int_{\text{one face}} E \cdot da = \frac{q}{24\varepsilon_0}.
\]