

Supplementary Assignments for the Logic & Sets Sections

We built formulas with logic operators \wedge , \vee , \neg , \rightarrow , \leftrightarrow , etc., and the constants T and F. In designing circuits, we can describe gates for only three operators: \wedge , \vee , \neg . Computer hardware engineer might want to make as few kinds of gates as possible. A set of logic operators is called complete if every well-formed formula of propositional logic is equivalent to a well-formed formula using the operators from the set. In fact, the set $\{\wedge, \vee, \neg\}$ is a complete set.

- 1). Do we need a \rightarrow gate for a circuit? Show your reasoning.
- 2). Find a formula equivalent to $a \rightarrow (b \wedge c \wedge d)$ using only the operators \wedge and \neg (and the constants T and F).
- 3). Show that the set $\{\neg, \wedge\}$ of operators is complete; and the set $\{\text{NAND}\}$ is also a complete set of operators.

(Hints: think along the line of logic equivalences).