Supplementary Assignments for the Proof and Induction Sections

1. A computational algorithm and the proof surrounding it:

The Square Root Algorithm produces an approximation of the square root of an integer by generating approximations that are alternately larger than the square root and then smaller than the square root. Each iteration of the procedure, however, brings the value of the approximation closer to the true value of the square root.

Square Root Algorithm: result – approximation of $\sqrt{17}$

```
Root = 4
for i = 1 to N do
        Root = (Root + 17/Root)/2
end
```

Now try to follow the algorithm by using N=4, and calculate the value of Root for each iteration and see how it gets closer to $\sqrt{17}$ each time.

With an iterative algorithm, it is important to know that the error gets smaller each iteration. Let R_n denote the value of Root after the for loop has been executed n times. Then, let $\varepsilon_n = \sqrt{17} - R_n$ is the error in the calculation after n executions of the for loop. The error could be either positive or negative.

Theorem: Prove that for the Square Root Algorithm, the error bound for R_n satisfies the inequality:

$$\left|\varepsilon_n\right| < (1/2)^{6\cdot 2^n - 3}$$

Hint: $\varepsilon_{n+1} = \sqrt{17} - R_{n+1}$, and $R_{n+1} = (R_n + 17/R_n)/2$, you can assume $R_n > 4$.

2. This is a writing / research type of assignment.

In the past century, many important theorems (or called conjectures) - which are significant for theoretical computer science and number theories - have been proved. For example, the Halting Problem addresses whether it is possible to determine a infinite loop (do not use Halting Problem for this assignment). Find a theorem of your interest, discuss the nature / format of the conjectures, and then discuss the validity of its proof methods.