Supplementary Assignments for the Recursion Sections

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1. Suppose that every hour 1% of a radioactive material decays (i.e., disappears). The process starts from 18 grams of such material.
a). Setup a recursive definition for the amount of this material after n hours.
b). What are the initial conditions for the recurrence relation.
c). Solve this recurrence equation.

2. The Fibonacci Numbers and Golden Ratio:
The Golden Ratio is a number built into many things in nature, such as the spiral of a nautilus sea shell, the disposition of leaves (or petals or seeds) in some plants like sunflowers, rabbit breeding patterns, and many geometric shapes. The Golden Ratio (or Number or Mean) is known under many names such as Phi and Divine Proportion. The Golden Ratio is said to be the most economical way to harmoniously arrange living volumes, with less effort and maximum efficiency. Since it exists in nature the number was not invented but rather discovered by many through the ages, but it is most frequently attributed to Leonardo Pisano, better know by his pen-name Fibonacci. He was a twelfth century Italian mathematician and his studies and writings popularized the ratio and its properties. Some say he discovered the ratio while modeling rabbit population growth, others say he was studying the Great Pyramid of

Gizeh. The Golden Ratio was, and still is, used extensively in architecture and art, and stock market investments.

The Fibonacci numbers are simply based on the sequence of numbers in which each successive number is the sum of the two previous ones:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, etc.

As the numbers get larger, the ratio between two successive numbers in the series approaches 0.618 (try it: 233 / 377 for example). The numbers satisfy the equation:

$$\mathbf{F_n = F_{n-1} + F_{n-2} \text{ with } F_1 = F_2 = 1}$$

and the mathematicians find the actual value of 0.618 (with thousands of decimals mind you) by solving the equation:

$$\mathbf{x = 1 + 1/x} \qquad (1)$$

Mathematically, the golden ratio is defined as the number:

$$x = \lim_{n\to\infty} \frac{F_n}{F_{n-1}} \qquad (2)$$

a). Now assume the number defined by equation (2) (golden ratio) exits, prove that it satisfy equation (1). (Hint, use the recursive definition of Fibonacci number)
b). Using the solution to the recursive equation, find the smallest Fibonacci number greater than 1,000,000.