Logic, Sets

Section 1.6 Sets

Set Definition

- A ***set*** is a collection of objects or *elements* or *members*.
  - A set is said to *contain* its elements.
  - There must be an underlying universal set U, either specifically stated or understood.
- **Notation:**
  - x is a member of S or x is an element of S:

    $x \in S$
  - x is not an element of S:

    $x \notin S$

    $2 \in$ {5,-7, π, “algebra”, 2, 2.718}

    $8 \notin$ {p : p is a prime number}

Set Representation

- List the elements between braces:

  S = {a, b, c, d}={b, c, a, d, d}

  Note: listing an object more than once does not change the set. Ordering means nothing.

- Specification by predicates (propositional functions):

  S= {x| *P*(x)},

  S contains all the elements from U which make the predicate *P* true.

  $\{(x, y) : x, y \in R \wedge x^2+y^2 = 1\}$

- Brace notation with ellipses:

  S = { . . . , -3, -2, -1}, the negative integers.

Common Universal Sets

- R = reals
- N = natural numbers = {0,1, 2, 3, . . . }, the *counting* numbers
- Z = all integers = {. . , -3, -2, -1, 0, 1, 2, 3, 4, .. .}
- Z+ is the set of positive integers

Subsets & Null Set

- **Definition:** The set A is a *subset* of the set B, denoted A $\subseteq$ B, iff $\forall$x[x $\in$A$\rightarrow$x $\in$B]
- **Definition:** The *void* set, the *null* set, the *empty* set, denoted $\varnothing$, is the set with no members.

Note: the assertion $x \in \varnothing$ is always false. Hence

$$\forall x[x \in \varnothing \rightarrow x \in B]$$

is always true. Therefore, $\varnothing$ is a subset of every set.

Note: Any set is always a subset of itself.

Relations on Sets

- **Definition:** If A $\subseteq$ B but A $\neq$ B the we say A is a *proper* subset of B, denoted $A \subset B$ (in some texts).
- **Definition:** The set of all subset of a set A, denoted P(A), is called the *power set* of A.
  - Example: If A = {a, b} then P(A) =
  - Example: P{1, 3, 5} =

More on Sets

- **Definition:** The number of (distinct) elements in A, denoted |A|, is called the *cardinality* of A.
  - If the cardinality is a natural number (in N), then the set is called *finite*, else *infinite*.
- Example:

  A = {a, b},

  |{a, b}| = 2,

  |P({a, b})| =

  |P{1, 3, 5}| =

More on Sets

- **Definition:** The *Cartesian product* of A with B, denoted A×B, is the set of ordered pairs {<a, b> | *a* ∈*A* ∧ *b*∈*B*}
- Notation:

$$\overset{n}{\underset{i=1}{X}} A_i = \{< a_1, a_2, \cdots, a_n >| a_i \in A_i\}$$

- Example:

  A = {a,b}, B = {1, 2, 3}

  AxB = {<a, 1>, <a, 2>, <a, 3>, <b, 1>, <b, 2>, <b, 3>}

  What is BxA?
- Example: if |A| = m and |B| = n, what is |AxB|?