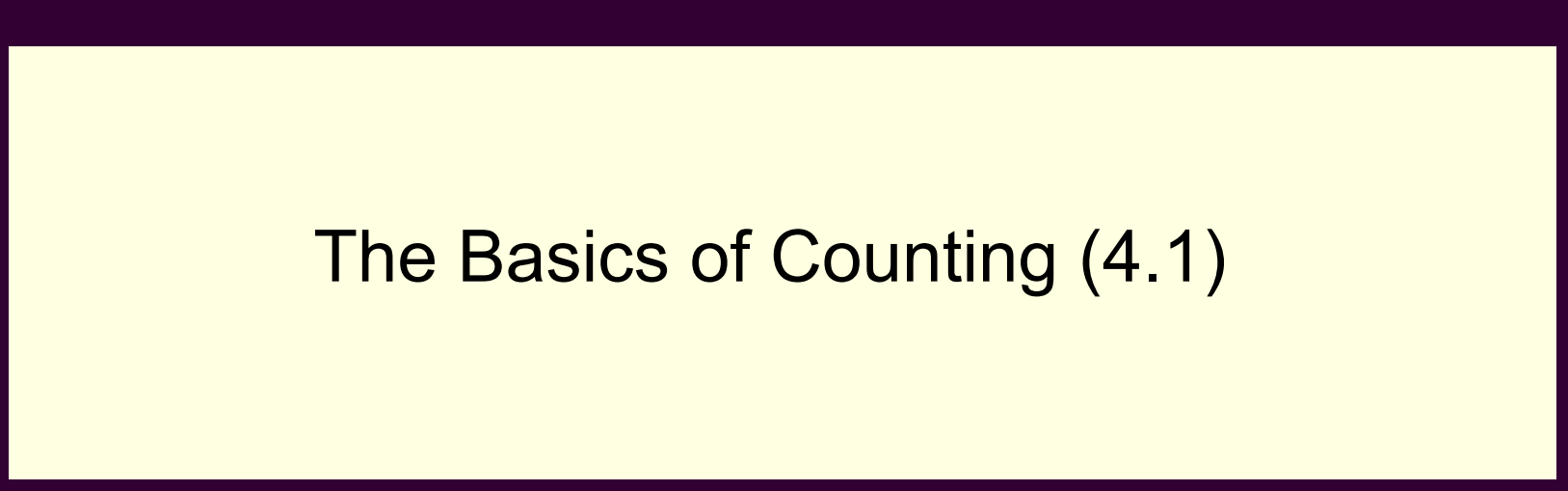




Counting Techniques



The Basics of Counting (4.1)

Why Counting Techniques

- Determine the complexity of algorithm.
- Determine the number of password, telephone number, Internet addresses.
Determine the probability of some events.
- The possible arrangement, cases for computer simulation.

The Sum Rule

- **The sum rule:** if a first task can be done in n_1 way and a second task can be done in n_2 way, and if these tasks can not be done at the same time, then there are n_1+n_2 ways to do either task.
- In the form of set: if A and B are disjoint, then $|A \cup B|=|A|+|B|$.
- The sum rule can be extended to n sets.

Examples

- A student can choose a computer project from one of three lists. The three lists contain 10, 20, 30 possible projects. There is no overlap among that list. How many projects are there to choose from?
- Suppose statement labels in a programming language must be a single letter or a single decimal digit. What is the number of possible labels?

The Product Rule

- **The product rule:** suppose that a procedure can be broken down into two tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are $n_1 n_2$ ways to do the procedure.
- In the form of set operation:
 $|A \times B| = |A| |B|$
- The product rule can be extended to n sets.

Examples

- A typical PIN is a sequence of any four symbols chosen from the 26 letters and the ten digits. How many different PINs are possible if repetition is allowed? What if repetition is not allowed?
- The ASCII character set is represented by 7 binary bits. How many character are there in the set?
- Count the number of binary bit strings of length 4 or less.

Summary

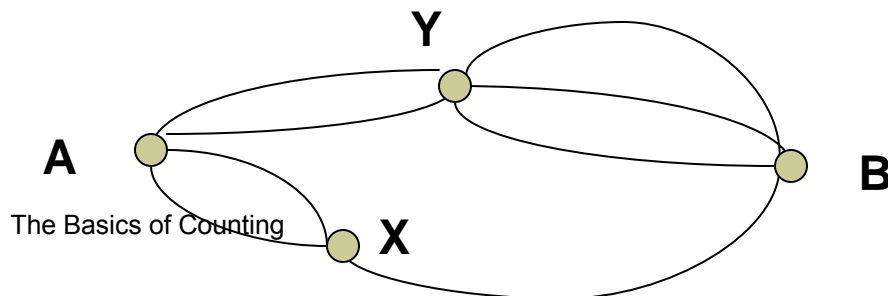
- The sum or product rules: which one to use?
 - If we need to make one choice and then another choice, use the product rule.
 - If we need to make one choice or another choice, use the sum rule.

Exercises

- There are 20 math majors and 100 computer majors at a college.
 - How many ways are there to pick two representative, so that one is a math major and the other is a computer major?
 - How many ways are there to pick one representative who is either a math major or a computer major?
- Seven-segment LEDs are used to display a digit or letter. How many different characters can be formed?

A Counting Strategy

- Set decomposition principle
 - Breaking the the problem into sub-problems; The solution of each may use the sum rule, the product rule, or combination of both.
 - The last step usually involve sum rule.
- Example: In the network setup bellow, what is the number of possible routing paths from point A to B through x or y.



Examples

Statement labels in Basic can be either a single letter or a letter followed by a decimal digit. Find the number of valid labels.

Solution:

We can partition the set of all labels L into the disjoint subsets consisting of:

- the set of single letter labels S
- the set of single letters followed by a digit D

$$L = S \cup D$$

IP Address Example

- Some facts about the Internet Protocol, version 4:
 - Valid computer addresses are in one of 3 types:
 - A *class A* IP address contains a 7-bit “netid” $\neq 1^7$, and a 24-bit “hostid”
 - A *class B* address has a 14-bit netid and a 16-bit hostid.
 - A *class C* addr. Has 21-bit netid and an 8-bit hostid.
 - The 3 classes have distinct headers (0, 10, 110)
 - Hostids that are all 0s or all 1s are not allowed.
- How many valid computer addresses are there?

The Rule of Inclusion-Exclusion

- If a first task can be done in n_1 way and a second task can be done in n_2 way, and if these tasks can be done at the same time (n_3 : the number of way to do both task at the same time), then there are $n_1+n_2-n_3$ to do either task.
- If A and B are not disjoint:
 $|A \cup B| = |A| + |B| - |A \cap B|$
- Can be viewed as a generalization of the sum rule.

Examples

Count the number of binary bit strings of length 4 which begin with a 1 or end with a 00.

Solution:

The set can be expressed as the union of:

- the subset S of strings which begin with 1
- the subset O that end in 00.

...

Examples

Count the set S of 3 digit numbers which begin or end with an even digit. Assume that 0 is even but a number cannot begin with 0.