6. Inductance and Capacitance, and Mutual Inductance

Two new circuit elements

Inductor

An inductor is an electrical component that opposes any change in electrical current. It is composed of a coil of wire wound around a supporting core. An inductor can store energy.

Capacitor

A capacitor is an electrical component that consists of two conductors separated by an insulator or dielectric material. A capacitor can store electrical charge.

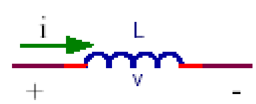
Inductors and capacitors are classified as passive elements and they cannot generate energy.

6.1 Inductor

The symbol for impedance is L and measured in henrys [H]. The relationship between the voltage and current at the terminals of an inductor:

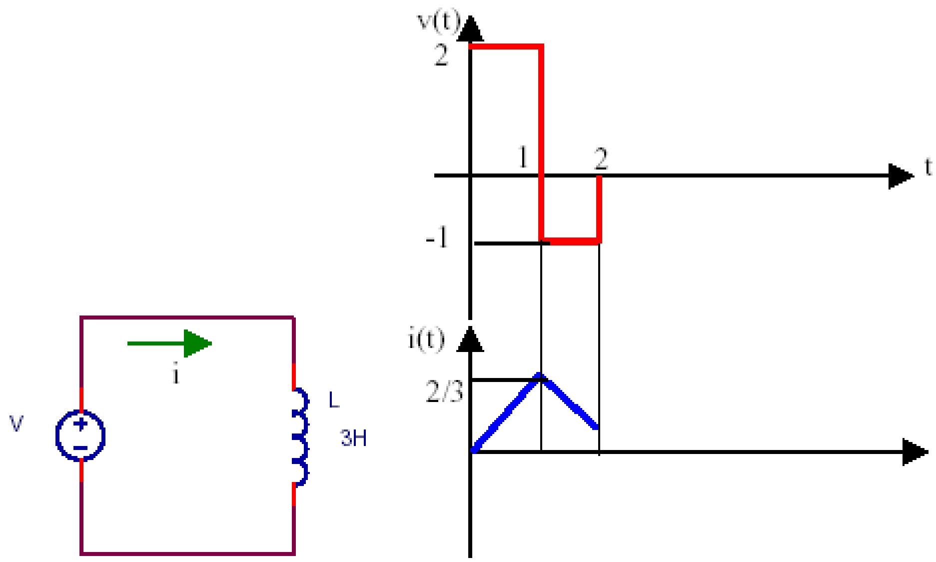
$$v = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$



- If current is constant, the voltage across the ideal inductor is zero
- Current cannot change instantaneously in the inductor

Example 6.1a



For t<0, i(t)=0

For 0<t≤1,

$$i(t) = \frac{1}{3}\int_0^t 2d\tau + i(0) = \frac{2}{3}t$$

For 1<t≤2,

$$i(t) = \frac{1}{3}\int_1^t (-1)d\tau + i(1) = -\frac{1}{3}t + 1$$

Power and Energy in the Inductor

If the current reference is in the direction of the voltage drop, the power:

$$p = vi = Li\frac{di}{dt}$$

In term of voltage,

$$p = v(\frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0))$$

Power is the rate of energy change,

$$p = \frac{dw}{dt} = Li\frac{di}{dt}$$

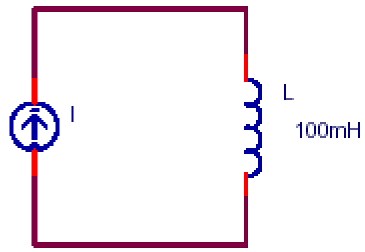
Thus,

$$dw = Li\,di$$

Integration on both sides,

$$w = \frac{1}{2}Li^2$$

Example 6.1b



$$i(t) = 0, t < 0$$

$$i(t) = 10te^{-5t}, t > 0$$

Find the voltage, power of the inductor. When the current reaches its maximum?

The maximum value of current is:

$$\frac{di}{dt} = 10(-5te^{-5t} + e^{-5t}) = 0 \rightarrow t = 0.2s$$

Voltage on the inductor

$$v(t) = L\frac{di}{dt} = (0.1)10(-5te^{-5t} + e^{-5t}) = e^{-5t}(1-5t)V, t > 0$$

$$p(t) = vi = 10te^{-10t} - 50t^2e^{-10t}W$$

$$w = \int_0^t p\,d\tau$$

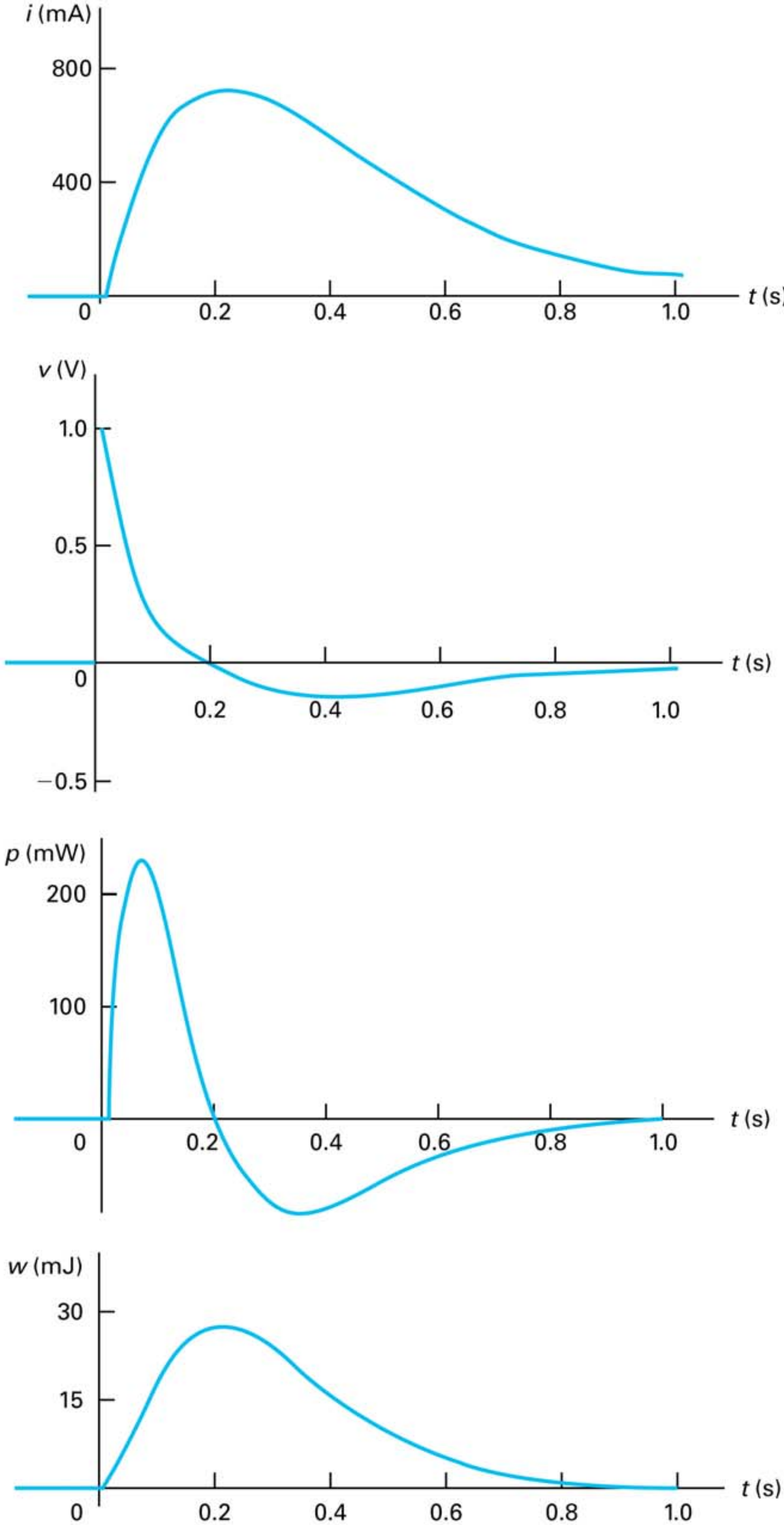
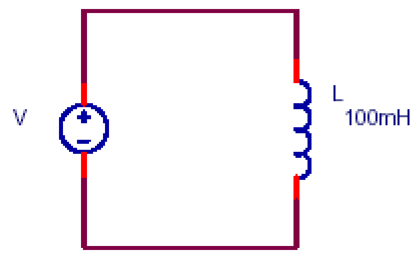


Figure 6.8 The variables i, v, p, and w versus t for Example 6.1.

From: Nilsson/Riedel, Electric Circuits, 6e, July 2000 Prentice Hall, Inc.

Example 6.3



$$v(t) = 0, t < 0$$

$$v(t) = 20te^{-10t}V, t > 0$$

Find i, p, w.

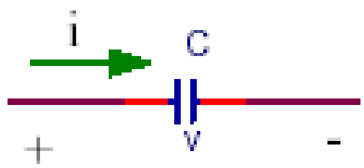
$$i(t) = \frac{1}{0.1}\int_0^t 20\tau e^{-10\tau} d\tau + 0 = 200[\frac{-e^{-10\tau}}{100}(10\tau + 1)]_0^t = 2(1 - 10te^{-10t} - e^{-10t})$$

$$p = vi = 20te^{-10t}[2(1 - 10te^{-10t} - e^{-10t})]W$$

$$w = \int_0^t p d\tau$$

6.2 Capacitor

It is represented by C and is measured in farads (F). The relationship between the voltage and current at the terminals of a capacitor:



$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$$

The power is:

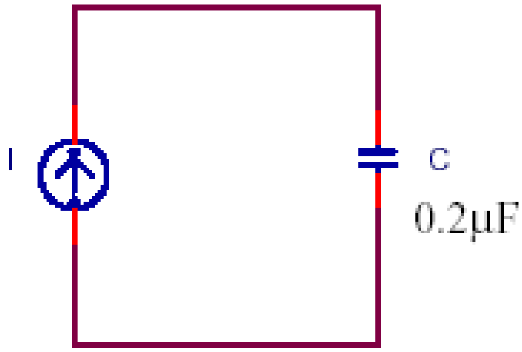
$$p = vi = Cv\frac{dv}{dt}$$

The energy is:

$$w = \frac{1}{2}cv^2$$

- A capacitor does not permit an instantaneous change in its terminal voltage.
- If voltage is constant, a capacitor behaves as an open circuit.
- Only a time-varying voltage can produce a displacement current.

Example 6.4



$$i(t) = \begin{cases} 0 & t < 0 \\ 5000tA & 0 \le t < 20\mu s \\ 0.2 - 5000tA & 20 \le t < 40\mu s \\ 0 & t \ge 40\mu s \end{cases}$$

Voltage:

For t≤0, *v=0*

For 0<t≤20µs,

$$v(t) = 5 \cdot 10^6 \int_0^t 5000\tau d\tau + 0 = (12.5)10^9 t^2 V$$

For 20<t≤40µs

$$v(t) = 5 \cdot 10^6 \int_{20}^t (0.2 - 5000\tau) d\tau + 5 = (10^6 t - (12.5)10^9 t^2 - 10)V$$ F

or t ≥ 40 µs

v(t)=v(40µs)=10V

Power:

For $t \le 0$, *p=?*

For $0 < t \le 20 \mu s$,

$$p(t) = iv = 5000t(12.5)10^9 t^2 = (62.5)10^{12} t^3 W$$

For $20 < t \le 40 \mu s$

p(t)=?

For $t \ge 40$ μs

P(t)=?

Energy:

For $t \le 0$, *w(t)=0*

For $0 < t \le 20 \mu s$,

$$p(t) = \frac{1}{2} cv^2 = (15.625)10^{12} t^4 J$$

For $20 < t \le 40 \mu s$

w(t)=?

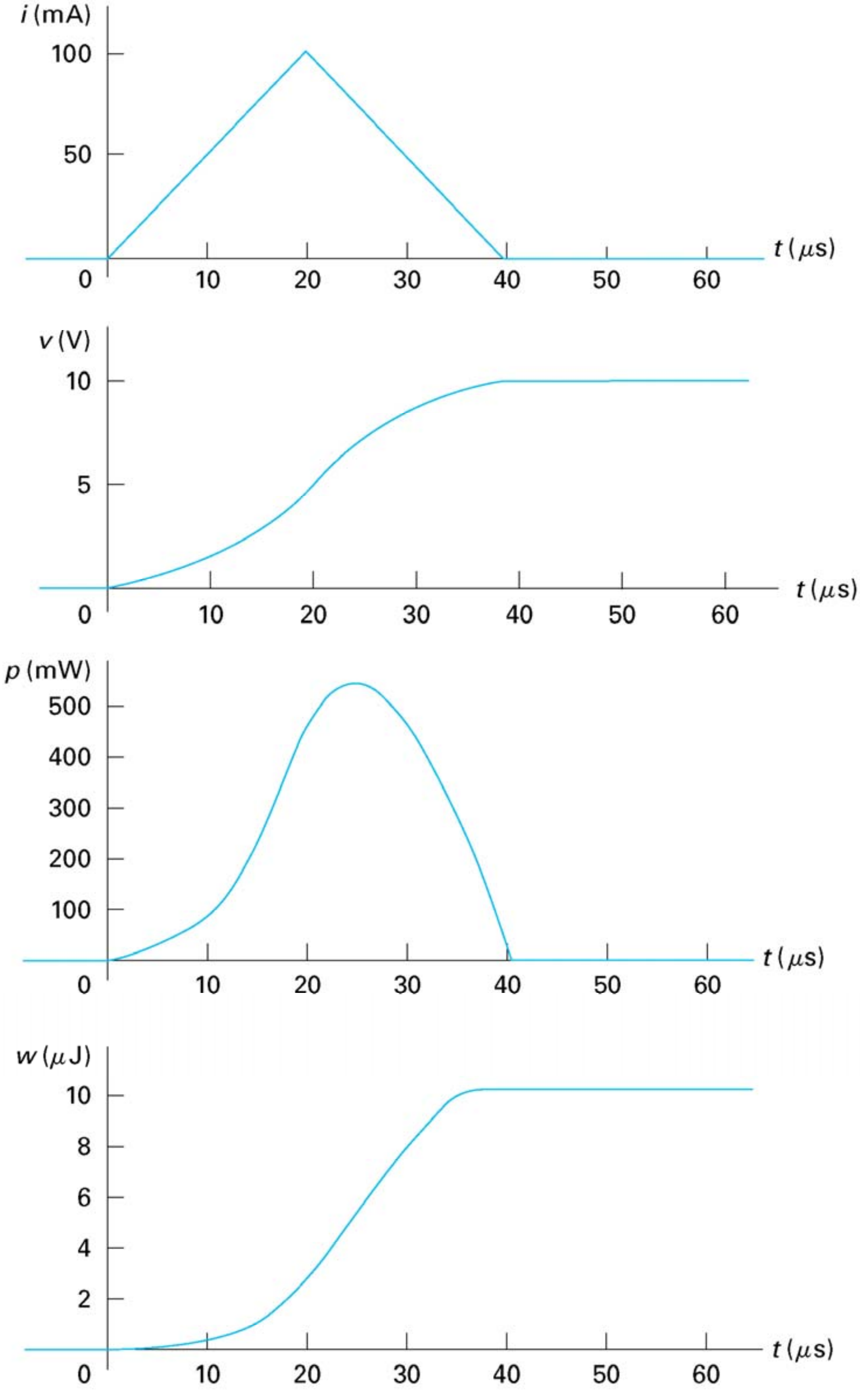
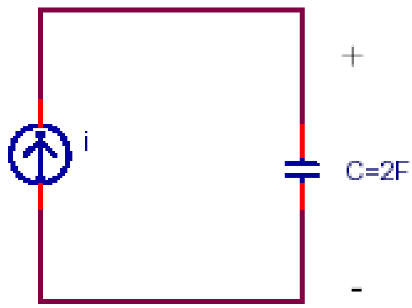


Figure 6.12 The variables i, v, p, and w versus t for Example 6.5.

Example 6.5



$$i(t) = \begin{cases} 0 & t < 0 \\ t & 0 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$

For t<0

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = 0$$

For 0≤ *t*<*2*

$$v(t) = v(0) + \frac{1}{C}\int_{0}^{t} i(\tau)d\tau = 0 + \frac{1}{2}\int_{0}^{t} \tau d\tau = \frac{1}{2}(\frac{t^2}{2})\mid_0^t = \frac{t^2}{4}$$

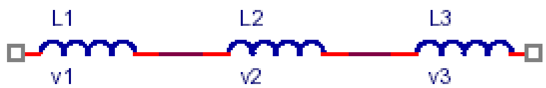
For t≥2

$$v(t) = v(2) + \frac{1}{C}\int_{2}^{t} i(\tau)d\tau = \frac{2^2}{4} + \frac{1}{2}\int_{2}^{t} 0 d\tau = 1$$

6.3 Serial-parallel combination of inductance and capacitance

Serial-parallel combination of inductors or capacitors can be reduced to a single inductor and capacitor.

Inductor is serial

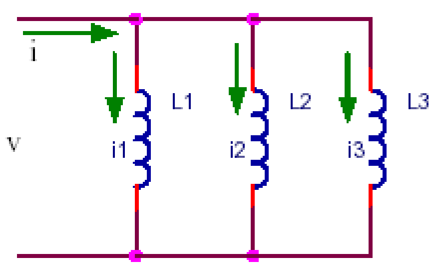


$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$L = L_1 + L_2 + L_3$$

$$v = L\frac{di}{dt}$$

Inductors in parallel



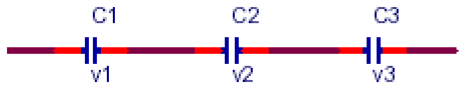
Total inductance value

$$i = i_1 + i_2 + i_3 = (\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3})\int_{t_0}^{t} v d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0)$$

$$i = \frac{1}{L}\int_{t_0}^{t} v d\tau + i(t_0)$$

$$\frac{1}{L} = (\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}), \qquad i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

Capacitors in serial

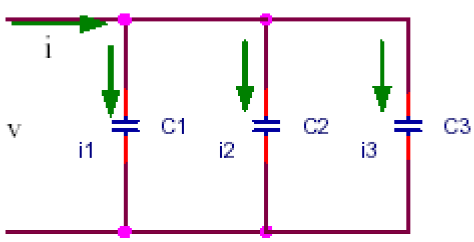


Total capacitor value:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

Capacitors in parallel



Total capacitor value:

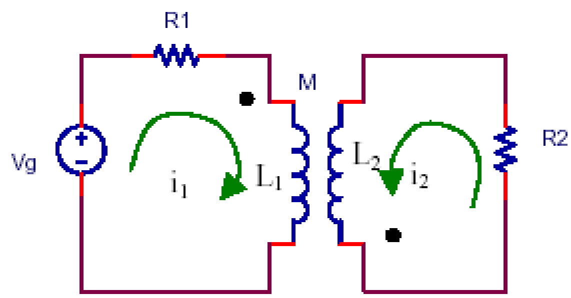
$$C = C_1 + C_2 + C_3$$

$$i(t) = C\frac{dv(t)}{dt}$$

6.4 Mutual Inductance

Inductance/self-inductance: is the parameter that relates a voltage to a time-varying current in the same circuit.

Mutual inductance: Two circuits are linked by a magnetic field. If current is time varying in the first circuit, it will be resulting magnetic field. This magnetic field induces voltage in the second circuit that related to current in the first circuit by parameter mutual inductance which is labeled *M*. Two coils are labeled L_1 and L_2.



Let's write the mesh currents equation:

$$-v_g + i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0$$

$$i_2 R_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$

Dot convention: used to keep track the polarity of induced voltage. When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.

Example: