Section 4.3 Permutations and Combinations

General techniques for counting the unordered selections of distinct objects and the ordered arrangements of objects of a finite set.

Permutation: is an ordered arrangement of distinct objects of a set.

An ordered r-selection from a set S ("traditionally" named: r-permutation) is a sequence of r objects from S.

A permutation is an arrangement that order matters.

After selecting the objects, two different orderings or arrangements constitute different permutations.

To compute r-permutation of a set with n element P(n, r):

- Choose the first object n ways,

- Choose the second object (since selection is without replacement) (n - 1) ways,

- …

- the r_{th} object (n - r + 1) ways.

By the rule of product, the number of permutations of n things taken r at a time,

$P(n,r) = n(n - 1)(n - 2) \ldots (n - r + 1)$

Note:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Example:

Suppose a salesman has to visit 8 different cities. He has to begin his trip in a specified city and end his trip in a specified city. He can visit the other cities in any order he wishes. How many possible orders can he use?

Combinations:

An **r-combination** of elements of a set is an unordered selection of r elements from the set. It is simply a subset of the set with r elements.

It is equivalent to selecting subsets of size r from a set of size n. Denoted as C(n, r).

The r permutation P(n, r) of the set can be obtain by:

1). Form the r-combinations C(n, r).

2). Ordering the r elements within each r-combination set, which can be done in P(r, r) way.

Thus, P(n, r)=C(n, r)P(r, r), and

$$C(n,r) = \binom{n}{r} = \frac{P(n,r)}{P(r,r)} = \frac{n!}{(n-r)!r!}$$

Example:

How many subsets of size r can be constructed from a set of n objects?

Example:

How many ways are there to select 5 players from a 10-member tennis team to make a trip to match at another school?

Example:

How many ways are there to select a committee to develop a discrete mathematics course if the committee is to consist of 3 faculty from the math department and 4 form computer department, if there are 9 faculty members in math department and 11 members in computer department.

Corollary:

C(n, r) = C(n, n-r)

Corollary:

$$\sum_{r=0}^{n} C(n,r) = 2^n$$

Example:

Suppose you flip a fair coin n times. How many different ways can you get

- no heads? C(n, 0)

- exactly one head? C(n, 1)

- exactly two heads? C(n, 2)

- exactly r heads? C(n, r)

- at least 2 heads?

Pascal's Identity:

C(n +1, k) = C(n, k-1) + C(n, k)

It produces Pascal’s triangle.

Example:

How many bit string (consisting of 0s and 1s) of length 4 have exactly 2 ones (or exactly 2 zeros)?

Example:

How many bit string of length 4 have at least 2 ones?