**T Flip-Flop**

A T (toggle) flip-flop is a complementing flip-flop and can be obtained from a JK flip-flop when the two inputs are tied together.

When \( T = 0 \) \( \rightarrow \) \( D = T \oplus Q = Q \)
\( D = Q \) and no change in output

When \( T = 1 \) \( \rightarrow \) \( D = T \oplus Q = Q' \)
\( D = Q' \) and the output complements

![T Flip-Flop](image)

**Characteristic Tables and Equations**

<table>
<thead>
<tr>
<th>( J )</th>
<th>( K )</th>
<th>( Q(t+1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( Q(t) ) No change</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0        Reset</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1        Set</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( Q'(t) ) Complement</td>
</tr>
</tbody>
</table>

\( Q(t + 1) = JQ' + K'Q \)

<table>
<thead>
<tr>
<th>( D )</th>
<th>( Q(t+1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0        Reset</td>
</tr>
<tr>
<td>1</td>
<td>1        Set</td>
</tr>
</tbody>
</table>

\( Q(t + 1) = D \)

\( Q(t) = \) present state
\( Q(t+1) = \) next state after one clock period

\( T \) | \( Q(t+1) \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( Q(t) ) No change</td>
</tr>
<tr>
<td>1</td>
<td>( Q'(t) ) Complement</td>
</tr>
</tbody>
</table>

\( Q(t + 1) = T \oplus Q = TQ' + T'Q \)
Analysis of Clocked Sequential Circuits

The behavior of a clocked sequential circuit is determined from the inputs, outputs, and the state of its flip-flops.

- **State Equation**
  A state equation (transition equation) specifies the next state as a function of the present state and inputs.

- **State Table**
  A state table (transition table) consists of: present state, input next state and output.

- **State Diagram**
  The information in a state table can be represented graphically in a state diagram. The state is represented by a circle and the transitions between states are indicated by directed lines connecting the circles.

Analysis Procedure

1. Determine the flip-Flop input equations in terms of the present state and input variables.

2. Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.

3. Use the corresponding state equations to determine the next state values in the state table.
Example of a Sequential Circuit

**State Equations**

\[ A(t+1) = A(t)x(t) + B(t)x(t) \]
\[ B(t+1) = A'(t)x(t) \]
\[ y = [A(t) + B(t)]x'(t) \]

\((t+1)\rightarrow\text{next state of the flip-flop one clock edge later.}\)

\[ A(t+1) = Ax + Bx \]
\[ B(t+1) = A'x \]
\[ y = (A + B)x' \]

Flip-flop input equations (excitation equations)

\[ D_A = Ax + Bx \]
\[ D_B = A'x \]

Example of a Sequential Circuit (continued)

\[ A(t+1) = Ax + Bx \]
\[ B(t+1) = A'x \]
\[ y = (A + B)x' \]

<table>
<thead>
<tr>
<th>Present State</th>
<th>Input</th>
<th>Next State</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A  B</td>
<td>x</td>
<td>A  B</td>
<td>y</td>
</tr>
<tr>
<td>0  0</td>
<td>0</td>
<td>0  0</td>
<td>0</td>
</tr>
<tr>
<td>0  0</td>
<td>1</td>
<td>0  1</td>
<td>0</td>
</tr>
<tr>
<td>0  1</td>
<td>0</td>
<td>1  1</td>
<td>1</td>
</tr>
<tr>
<td>0  1</td>
<td>1</td>
<td>1  1</td>
<td>0</td>
</tr>
<tr>
<td>1  0</td>
<td>0</td>
<td>0  0</td>
<td>1</td>
</tr>
<tr>
<td>1  0</td>
<td>1</td>
<td>1  0</td>
<td>0</td>
</tr>
<tr>
<td>1  1</td>
<td>0</td>
<td>0  0</td>
<td>1</td>
</tr>
<tr>
<td>1  1</td>
<td>1</td>
<td>1  1</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 5-15 Example of Sequential Circuit

*note mistake in Fig. 5-15 p. 181*
### Example of a Sequential Circuit (continued)

<table>
<thead>
<tr>
<th>Present State</th>
<th>Input</th>
<th>Next State</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A  B</td>
<td>x</td>
<td>A  B</td>
<td>y</td>
</tr>
<tr>
<td>0  0</td>
<td>0</td>
<td>0  0</td>
<td>0</td>
</tr>
<tr>
<td>0  1</td>
<td>0</td>
<td>1  0</td>
<td>0</td>
</tr>
<tr>
<td>0  0</td>
<td>1</td>
<td>0  1</td>
<td>0</td>
</tr>
<tr>
<td>0  0</td>
<td>1</td>
<td>1  1</td>
<td>0</td>
</tr>
<tr>
<td>1  0</td>
<td>0</td>
<td>0  0</td>
<td>1</td>
</tr>
<tr>
<td>1  0</td>
<td>1</td>
<td>1  1</td>
<td>0</td>
</tr>
<tr>
<td>1  1</td>
<td>0</td>
<td>0  0</td>
<td>1</td>
</tr>
<tr>
<td>1  1</td>
<td>1</td>
<td>1  1</td>
<td>1</td>
</tr>
</tbody>
</table>

**a**: When the sequential circuit is in present state 00 and the input is 1, the output is 0. After the next clock cycle, the circuit goes to the next state 01.

**b**: When the sequential circuit is in present state 01 and the input is 1, the output is 0. After the next clock cycle, the circuit goes to the next state 11.

**c**: No change in state.

---

### Sequential Circuit Analysis with $D$ Flip-Flops

\[ D_A = A \oplus x \oplus y \]

\[ A(t+1) = A \oplus x \oplus y \]

(a) Circuit diagram

(b) State table

<table>
<thead>
<tr>
<th>Present state</th>
<th>Inputs</th>
<th>Next state</th>
</tr>
</thead>
<tbody>
<tr>
<td>A  x  y  A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0  0  0  0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0  0  1  1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0  1  0  1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0  1  1  0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1  0  0  1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1  0  1  0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1  1  0  0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1  1  1  1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) State diagram

Fig. 5-16 State Diagram of the Circuit of Fig. 5-15

Fig. 5-17 Sequential Circuit with $D$ Flip-Flop
Example of Sequential Circuit with \( JK \) Flip-Flops

1. Flip-Flop input equations:

\[
\begin{align*}
J_A &= B & K_A &= Bx' \\
J_B &= x' & K_B &= A \oplus x
\end{align*}
\]

![ JK FF circuit diagram ](Fig. 5-18 Sequential Circuit with JK Flip-Flop)

2. Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.

\[
\begin{align*}
A(t+1) &= J_A A + K_A' A \\
B(t+1) &= J_B B' + K_B' B
\end{align*}
\]

**JK Flip-Flop characteristic equation**

**Flip-Flop input equations**

\[
\begin{align*}
J_A &= B & K_A &= Bx' \\
J_B &= x' & K_B &= A \oplus x
\end{align*}
\]

**Sequential Circuit state equations**

\[
\begin{align*}
A(t+1) &= B A' + (Bx')' A = A'B + AB' + Ax \\
B(t+1) &= x' B' + (A \oplus x)' B = B'x' + ABx + A'Bx'
\end{align*}
\]

3. Use the corresponding state equations to determine the next state values in the state table.
Example of Sequential Circuit with \( JK \) FF (continued)

<table>
<thead>
<tr>
<th>Present State</th>
<th>Input</th>
<th>Next State</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) ( B )</td>
<td>( x )</td>
<td>( A ) ( B )</td>
</tr>
<tr>
<td>0 0 0 0 0 1</td>
<td>0 0 0 1 0 0</td>
<td></td>
</tr>
<tr>
<td>0 0 1 0 0 0</td>
<td>0 0 1 1 0 0</td>
<td></td>
</tr>
<tr>
<td>0 1 0 1 1 0</td>
<td>0 1 1 1 1 0</td>
<td></td>
</tr>
<tr>
<td>0 1 1 1 0 0</td>
<td>0 1 1 0 0 1</td>
<td></td>
</tr>
<tr>
<td>1 0 0 1 1 0</td>
<td>0 0 1 0 0 1</td>
<td></td>
</tr>
<tr>
<td>1 0 1 1 1 0</td>
<td>0 0 1 0 0 0</td>
<td></td>
</tr>
<tr>
<td>1 1 0 0 0 0</td>
<td>1 1 1 0 0 1</td>
<td></td>
</tr>
<tr>
<td>1 1 1 1 1 1</td>
<td>1 0 0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
A(t+1) &= BA' + (Bx')' A = A' B + AB' + A \times x \\
B(t+1) &= x' B' + (A \oplus x)' B = B' x' + ABx + A' Bx'
\end{align*}
\]

Example of Sequential Circuit with \( T \) Flip-Flops

1. Flip-Flop input equations:

\[
\begin{align*}
T_A &= Bx \\
T_B &= x \\
y &= AB
\end{align*}
\]
Example: \( T \) Flip-Flops circuit (continued)

2. Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.

\[
Q(t+1) = T \oplus Q = T'Q + Q'T
\]

\[
\begin{align*}
T_A &= Bx \\
T_B &= x \\
y &= AB
\end{align*}
\]

Flip-Flop input equations

\[
A(t+1) = (Bx)'A + Bx(A') = AB' + Ax' + A'Bx \\
B(t+1) = x \oplus B
\]

Sequential Circuit state equations

<table>
<thead>
<tr>
<th>Present State</th>
<th>Input</th>
<th>Next State</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) ( B )</td>
<td>( x )</td>
<td>( A ) ( B )</td>
<td>( y )</td>
</tr>
<tr>
<td>0 0</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 0</td>
<td>1</td>
<td>0 1</td>
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</tr>
<tr>
<td>1 1</td>
<td>1</td>
<td>0 0</td>
<td>1</td>
</tr>
</tbody>
</table>

Example: \( T \) Flip-Flops circuit (continued)

\[
A(t+1) = (Bx)'A + Bx(A') = AB' + Ax' + A'Bx \\
B(t+1) = x \oplus B
\]