Digital vs. Analog

- An analog system has continuous range of values
  - A mercury thermometer
  - Vinyl records
  - Human eye

- A digital system has a set of discrete values
  - Digital Thermometer
  - Compact Disc (CD)
  - Digital camera

Benefits of using digital

- Cheap electronic circuits
- Easier to calibrate and adjust
- Resistance to noise: Clearer picture and sound

Binary System

- Discrete elements of information are represented with bits called binary codes.

Example: $(09)_{10} = (1001)_2$
$(15)_{10} = (1111)_2$

Question: Why are commercial products made with digital circuits as opposed to analog?

Most digital devices are programmable: By changing the program in the device, the same underlying hardware can be used for many different applications.
Review the decimal number system.
Base (Radix) is 10 - symbols (0,1, .. 9) Digits
For Numbers > 9, add more significant digits in position to the left, e.g. 19>9.
Each position carries a weight.

<table>
<thead>
<tr>
<th>Weight</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^9</td>
<td>1</td>
</tr>
<tr>
<td>10^8</td>
<td>1</td>
</tr>
<tr>
<td>10^7</td>
<td>1</td>
</tr>
<tr>
<td>10^6</td>
<td>1</td>
</tr>
<tr>
<td>10^5</td>
<td>1</td>
</tr>
<tr>
<td>10^4</td>
<td>1</td>
</tr>
<tr>
<td>10^3</td>
<td>1</td>
</tr>
<tr>
<td>10^2</td>
<td>1</td>
</tr>
<tr>
<td>10^1</td>
<td>1</td>
</tr>
<tr>
<td>10^0</td>
<td>1</td>
</tr>
</tbody>
</table>

If we were to write 1936.25 using a power series expansion and base 10 arithmetic:

\[
1 	imes 10^3 + 9 	imes 10^2 + 3 	imes 10^1 + 6 	imes 10^0 + 2 	imes 10^{-1} + 5 	imes 10^{-2}
\]

Binary Code

- Review the decimal number system.
  Base (Radix) is 10 - symbols (0,1, .. 9) Digits
  For Numbers > 9, add more significant digits in position to the left, e.g. 19>9.
  Each position carries a weight.

<table>
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<tr>
<th>Weight</th>
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<tr>
<td>2^9</td>
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</tr>
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<td>2^5</td>
<td>1</td>
</tr>
<tr>
<td>2^4</td>
<td>1</td>
</tr>
<tr>
<td>2^3</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>2^1</td>
<td>1</td>
</tr>
<tr>
<td>2^0</td>
<td>1</td>
</tr>
</tbody>
</table>

If we write 10111.01 using a decimal power series we convert from binary to decimal:

\[
1 	imes 2^4 + 0 	imes 2^3 + 1 	imes 2^2 + 1 	imes 2^1 + 1 	imes 2^0 + 0 	imes 2^{-1} + 1 	imes 2^{-2}
= 1 	imes 16 + 0 	imes 8 + 1 	imes 4 + 1 	imes 2 + 1 	imes 1 + 0 	imes 0.5 + 1 	imes 0.25 = 23.25
\]

The binary number system.
- Base is 2 - symbols (0,1) - Binary Digits (Bits)
- For Numbers > 1, add more significant digits in position to the left, e.g. 10>1.
- Each position carries a weight (using decimal).

Octal/Hex number systems

- The octal number system [from Greek: OKTO].
  - Its base is 8 ⇒ eight digits 0, 1, 2, 3, 4, 5, 6, 7
  - In computer work: \(2^{10} = 1024\) is referred as K = kilo
  - \(2^{20} = 1048576\) is referred as M = mega

- The hexadecimal number system [from Greek: AEKAEZ].
  - Its base is 16 ⇒ first 10 digits are borrowed from the decimal system and the letters A, B, C, D, E, F are used for the digits 10, 11, 12, 13, 14, 15

\[
(110000.0111)_2 = \text{Ans: } 48.4375
\]

\[
(236.4)_{10} = (158.5)_{10}
\]

\[
5.15884868382 = 1 	imes 2^{-1} + 0 	imes 2^{-2} + 1 	imes 2^{-3} + 1 	imes 2^{-4} + 0 	imes 2^{-5} + 1 	imes 2^{-6}
\]

\[
(D63FA)_{16} = (877562)_{10}
\]

What is the exact number of bytes in a 16 Gbyte memory module?
Conversion from Decimal to Binary

Let each bit of a binary number be represented by a variable whose subscript = bit positions, i.e.,

\[(110)_2 = (a_2a_1a_0)_2\]

Its decimal equivalent is:

\[(1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0)_{10} = (a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0)_{10}\]

It is necessary to separate the number into an integer part and a fraction: Repeatedly divide the decimal number by 2.

Conversion from Decimal to Binary

Find the binary equivalent of 37.

\[\begin{align*}
2 \div 37 &= 18 + 0.5 & \text{LSB} \\
2 \div 18 &= 9 + 0 & 0 \\
2 \div 9 &= 4 + 0.5 & 1 \\
2 \div 4 &= 2 + 0 & 0 \\
2 \div 2 &= 1 + 0 & 0 \\
2 \div 1 &= 0 + 0.5 & 1 \\
\end{align*}\]

\[37_{10} = 100101_2\]

\[53_{10} = \_\_\_\_\_\_ \text{ANS: } 53_{10} = 110101_2\]

Conversion from Decimal to Binary

Convert (0.8542)\(_{10}\) to binary (give answer to 6 digits).

\[\begin{align*}
0.8542 \times 2 &= 1 + 0.7084 & a_1 = 1 \\
0.7084 \times 2 &= 1 + 0.4168 & a_2 = 1 \\
0.4168 \times 2 &= 0 + 0.8336 & a_3 = 0 \\
0.8336 \times 2 &= 1 + 0.6672 & a_4 = 1 \\
0.6675 \times 2 &= 1 + 0.3344 & a_5 = 1 \\
0.3344 \times 2 &= 0 + 0.6688 & a_6 = 0 \\
\end{align*}\]

\[(0.8542)_{10} = (0.a_1a_2a_3a_4a_5a_6)_2 = (0.110110)_{2}\]

\[ (53.8542)_{10} = (\_\_\_\_\_\_\_\_)_2 \]

Conversion from Decimal to Octal

The decimal number is first divided by 8. The remainder is the LSB. The quotient is then divide by 8 and the remainder is the next significant bit and so on.

Convert 1122 to octal.

\[\begin{align*}
8 \div 1122 &= 140 + 0.25 & R2 \quad \text{LSB} \\
8 \div 140 &= 17 + 0.5 & R4 \\
8 \div 17 &= 2 + 0.125 & R1 \\
8 \div 2 &= 0 + 0.25 & R2 \quad \text{MSB} \\
\end{align*}\]

\[1122_{10} = 2142_8\]
**Conversion from Decimal to Octal**

- Convert \((0.3152)_{10}\) to octal (give answer to 4 digits).

  \[
  0.3152 	imes 8 = 2 + 0.5216 \quad a_1 = 2
  \]
  \[
  0.5216 	imes 8 = 4 + 0.1728 \quad a_2 = 4
  \]
  \[
  0.1728 	imes 8 = 1 + 0.3824 \quad a_3 = 1
  \]
  \[
  0.3824 	imes 8 = 3 + 0.0592 \quad a_4 = 3
  \]

- \((0.3152)_{10} = (0.a_1a_2a_3a_4)_{10} = (0.2413)_8\)

- \((1122.3152)_{10} = (\ ? )_8\)

**Conversion using Table**

- Conversion from and to binary, octal, and hexadecimal plays an important part in digital computers.

  - Since \(2^3 = 8\) and \(2^4 = 16\)
  - Each octal digit corresponds to 3 binary digits
  - Each hex digit corresponds to 4 binary digits.

- \((010 111 100 . 001 011 000)_{10} = (274.130)_8\)

- \((0110 1111 1101 . 0001 0011 0100)_{10} = (6FD.134)_{16}\)

**Complements**

- Complements: They are used in digital computers for subtraction operation and for logic manipulation.

  - 10’s complement and 9’s complement
  - 2’s complement and 1’s complement

  **Decimal Numbers**
  - 9’s complement of \(N = (10^N-1) - N\) \((N\ is\ a\ decimal\ #)\)
  - 10’s complement of \(N = 10^N - N\) \((N\ is\ a\ decimal\ #)\)

  **Binary Numbers**
  - 1’s complement of \(N = (2^N-1) - N\ \((N\ is\ a\ binary\ #)\)
  - 1’s complement can be formed by changing 1’s to 0’s and 0’s to 1’s
  - 2’s complement of a number is obtained by leaving all least significant 0’s and the first 1 unchanged, and replacing 1’s with 0’s and 0’s with 1 in all higher significant digits.

- \((0110 1111 1101 . 0001 0011 0100)_{10} = (6FD.134)_{16}\)

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Hex</th>
<th>Binary</th>
<th>Octal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00</td>
<td>0000</td>
<td>00</td>
</tr>
<tr>
<td>1</td>
<td>01</td>
<td>0010</td>
<td>02</td>
</tr>
<tr>
<td>2</td>
<td>02</td>
<td>0011</td>
<td>03</td>
</tr>
<tr>
<td>3</td>
<td>03</td>
<td>0100</td>
<td>04</td>
</tr>
<tr>
<td>4</td>
<td>04</td>
<td>0101</td>
<td>05</td>
</tr>
<tr>
<td>5</td>
<td>05</td>
<td>0110</td>
<td>06</td>
</tr>
<tr>
<td>6</td>
<td>06</td>
<td>0111</td>
<td>07</td>
</tr>
<tr>
<td>7</td>
<td>07</td>
<td>1000</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>08</td>
<td>1001</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>09</td>
<td>1010</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>0A</td>
<td>1011</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>0B</td>
<td>1100</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>0C</td>
<td>1101</td>
<td>15</td>
</tr>
<tr>
<td>13</td>
<td>0D</td>
<td>1110</td>
<td>16</td>
</tr>
<tr>
<td>14</td>
<td>0E</td>
<td>1111</td>
<td>17</td>
</tr>
<tr>
<td>15</td>
<td>0F</td>
<td>1111</td>
<td>18</td>
</tr>
</tbody>
</table>
### Complements

<table>
<thead>
<tr>
<th>9’s complement of $N = (10^n-1) - N$ ($N$ is a decimal #)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The 9’s complement of 12345 = (10^5 – 1) – 12345 = 87654</td>
</tr>
<tr>
<td>The 9’s complement of 012345 = (10^6 – 1) – 012345 = 987654</td>
</tr>
<tr>
<td>10’s complement of $N = [(10^n – 1) – N] + 1$ ($N$ is a decimal #)</td>
</tr>
<tr>
<td>The 10’s complement of 739821 = 10^6 – 739821 = 260179</td>
</tr>
<tr>
<td>The 10’s complement of 2500 = 10^4 – 2500 = 7500</td>
</tr>
<tr>
<td>Find the 9’s and 10’s-complement of 00000000</td>
</tr>
</tbody>
</table>

ANS: 99999999 and 00000000

### 1’s and 2’s Complements

<table>
<thead>
<tr>
<th>1’s complement of $N = (2^n-1) – N$ ($N$ is a binary #)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1’s complement can be formed by changing 1’s to 0’s and 0’s to 1’s</td>
</tr>
<tr>
<td>2’s complement of a number is obtained by leaving all least significant 0’s and the first 1 unchanged, and replacing 1’s with 0’s and 0’s with 1 in all higher significant digits.</td>
</tr>
<tr>
<td>The 1’s complement of 1101011 = 0010100</td>
</tr>
<tr>
<td>The 2’s complement of 0110111 = 1001001</td>
</tr>
<tr>
<td>Find the 1’s and 2’s-complement of 10000000</td>
</tr>
</tbody>
</table>

**Answer**: 01111111 and 10000000

### Subtraction Using Complements

<table>
<thead>
<tr>
<th>Subtraction with digital hardware using complements:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction of two $n$-digit unsigned numbers $M – N$ base $r$:</td>
</tr>
<tr>
<td>1. Add $M$ to the $r$’s complement of $N$: $M + (r^n – N)$</td>
</tr>
<tr>
<td>2. If $M \leq N$, the sum will produce an end carry and is equal to $r^n$ that can be discarded. The result is then $M – N$.</td>
</tr>
<tr>
<td>3. If $M \geq N$, the sum will not produce an end carry and is equal to $r^n – (N – M)$</td>
</tr>
</tbody>
</table>

### Decimal Subtraction using complements

<table>
<thead>
<tr>
<th>Subtract 150 – 2100 using 10’s complement:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 150$</td>
</tr>
<tr>
<td>10’s complement of $N = 7900$</td>
</tr>
<tr>
<td>Sum = 8050</td>
</tr>
<tr>
<td>There’s no end carry ➔ negative</td>
</tr>
<tr>
<td><strong>Answer</strong>: (10’s complement of 8050) = – 1950</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subtract 7188 – 3049 using 10’s complement:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 7188$</td>
</tr>
<tr>
<td>10’s complement of $N = 6051$</td>
</tr>
<tr>
<td>Sum = 14139</td>
</tr>
<tr>
<td><strong>Answer</strong>: 4139</td>
</tr>
<tr>
<td>Discard end carry 10^4 = – 10000</td>
</tr>
</tbody>
</table>
Subtract 1010100 – 1000011 using 2’s complement:

\[ A = 1010100 \]
\[ 2's \ complement \ of \ B = +0111101 \]
\[ Sum = \quad 1000000 \]
\[ \text{Discard end carry} = -1000000 \]
\[ Answer = \quad 0010001 \]

Binary subtraction is done using the same procedure.

Subtract 1000011 – 1010100 using 2’s complement:

Answer = –0010001

Binary Subtraction using complements

Subtract 1010100 – 1000011 using 1’s complement:

\[ A = 1010100 \]
\[ 1's \ complement \ of \ B = +0111100 \]
\[ Sum = \quad 1001000 \]
\[ \text{End-around carry} = +1 \]
\[ Answer = \quad 0010001 \]

Subtract 1000011 – 1010100 using 1’s complement:

Answer = –0010001

Signed binary numbers

To represent a negative binary number, the convention is to make the sign bit 1. [sign bit 0 is for positive]

<table>
<thead>
<tr>
<th>Binary</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>01001</td>
<td>9</td>
</tr>
<tr>
<td>+9 (signed)</td>
<td></td>
</tr>
<tr>
<td>11001</td>
<td>25</td>
</tr>
<tr>
<td>–9 (signed)</td>
<td></td>
</tr>
</tbody>
</table>

Arithmetic addition

Negative numbers must be initially in 2’s complement form and if the obtained sum is negative, it is in 2’s complement form.

\[
\begin{align*}
+6 & \quad 00000110 \quad -6 \quad 11111010 \\
+13 & \quad 00001101 \quad +13 \quad 00001101 \\
+19 & \quad 00010011 \quad +7 \quad 00000111 \\
\end{align*}
\]

Add –6 and –13

Answer = 11101101
Transfer of Information with Registers

Binary Information Processing

Fig. 1-1 Transfer of information with registers

Fig. 1-2 Example of binary information processing