Problem 1. Find the value of $k (k > 1)$ such that the region enclosed by $y = \frac{1}{2} \ln x$, $y = 0$, and $x = k$ has an area of 4 square units.

Problem 2. Let $f(x) = 2 + 3^{-x}, g(x) = 2^x$.

(a) Plot the two functions on the same coordinate system and estimate the point of intersection.

(b) Use appropriate command to find the point of intersection of the two functions and verify your answer.

Problem 3. Let assume that a rumor spreads according to the following equation

$$p(t) = \frac{2}{2 + ae^{-kt}}.$$  

Where $p(t)$ is the ratio of the population that knows the rumor at time $t$ and $a$ and $k$ are positive constants.

(i) Find the rate of the spread of the rumor.

(ii) Graph $p$ for the case $a = 8, k = .5$ with $t$ measures in hours. Estimate how long it will take for 80% of the population to hear the rumor.