Math 240
Fall 2006
Dr. Khalili

MAPLE PROJECT 3

Due date November.3, 2006

Problem 1. The harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots$$

is one of the most impotant series in chapter 8 (infinite series.) Note that the general term of the series, $\frac{1}{n}$, goes to zero as n goes to infinity, however the series diverges!
In other words, even though the terms are getting smaller and smaller, the sum "adds up to infinity."

(a) One way to show that the harmonic series diverges is due to **J. Bernoulli**. He grouped the terms of the harmonic series as follows:

$$1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{>\frac{1}{2}} + \underbrace{\frac{1}{5} + \cdots + \frac{1}{8}}_{>\frac{1}{2}} + \underbrace{\frac{1}{9} + \cdots + \frac{1}{16}}_{>\frac{1}{2}} + \underbrace{\frac{1}{17} + \cdots + \frac{1}{32}}_{>\frac{1}{2}} + \cdots$$

Write a short paragraph explaining how you can use this grouping to show that the harmonic series diverges.

(b) How many terms M you need so that

$$\sum_{n=1}^{M} \frac{1}{n} > 50$$

(c) Show that the sum of the first million terms of the harmonic series is less than 15.

(d) Show that the following inequalities are valid.

$$\ln\frac{21}{10} \le \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{19} + \frac{1}{20} \le \ln\frac{20}{9}$$

$$\ln\frac{201}{100} \le \frac{1}{100} + \frac{1}{101} + \frac{1}{102} + \cdots + \frac{1}{199} + \frac{1}{200} \le \ln\frac{200}{99}$$

(e) Find the following limit in two different ways (i) using the ideas in part (d) and (ii) using the limit command of MAPLE.

$$\lim_{m\to\infty} \sum_{n=m}^{2m} \frac{1}{n}$$