

MAPLE PROJECT 3
 Due date April.14, 2006

Problem 1. The harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots$$

is one of the most important series in chapter 8 (infinite series.) Note that the general term of the series, $\frac{1}{n}$, goes to zero as n goes to infinity, however the series diverges!

In other words, even though the terms are getting smaller and smaller, the sum “adds up to infinity.”

- (a) One way to show that the harmonic series diverges is due to **J. Bernoulli**. He grouped the terms of the harmonic series as follows:

$$1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{> \frac{1}{2}} + \underbrace{\frac{1}{5} + \cdots + \frac{1}{8}}_{> \frac{1}{2}} + \underbrace{\frac{1}{9} + \cdots + \frac{1}{16}}_{> \frac{1}{2}} + \underbrace{\frac{1}{17} + \cdots + \frac{1}{32}}_{> \frac{1}{2}} + \cdots$$

Write a short paragraph explaining how you can use this grouping to show that the harmonic series diverges.

- (b) How many terms M you need so that

$$\sum_{n=1}^M \frac{1}{n} > 50$$

- (c) Show that the sum of the first million terms of the harmonic series is less than 15.

- (d) Show that the following inequalities are valid.

$$\ln \frac{21}{10} \leq \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{19} + \frac{1}{20} \leq \ln \frac{20}{9}$$

$$\ln \frac{201}{100} \leq \frac{1}{100} + \frac{1}{101} + \frac{1}{102} + \cdots + \frac{1}{199} + \frac{1}{200} \leq \ln \frac{200}{99}$$

- (e) Find the following limit in two different ways (i) using the ideas in part (d) and (ii) using the limit command of MAPLE.

$$\lim_{m \rightarrow \infty} \sum_{n=m}^{2m} \frac{1}{n}$$