Dr. Khalili
Fall 2006

Math 579 Real Analysis
Project 1
Due October 20

The sequence of *Fibonacci numbers* is defined recursively

$$F_0 = 0, \; F_1 = 1, \quad F_{n+1} = F_n + F_{n-1}, \quad \text{for } n = 1, 2, 3, \cdots$$

Prove that

(a) $F_2 + F_4 + F_6 + \cdots + F_{2n} = F_{2n+1} - 1,$ for all $n \in \mathbb{N}$

(b) $\Sigma_{k=1}^{n} (F_k)^2 = F_n \, F_{n+1},$ for all $n \in \mathbb{N}$

(c) $F_{n-1} \, F_{n+1} - (F_n)^2 = (-1)^n$ for all $n \in \mathbb{N}$

(d) $F_1 - F_2 + F_3 - F_4 + \cdots + (-1)^n F_n = (-1)^{n+1} F_{n-1} + 1$ for all $n \in \mathbb{N}$