

Math 579 Real Analysis
Project 2
Due November 20

Use the following steps to show that the sequence

$$s_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} - \ln(n)$$

has a limit. (The value of the limit is denoted by γ and is called Euler's constant.)

(a) Draw a picture of $f(x) = \frac{1}{x}$ and interpret s_n as an area to show that $s_n > 0$ for all n .

(b) Interpret

$$s_n - s_{n+1} = [\ln(n+1) - \ln(n)] - \frac{1}{n+1}$$

as a difference of areas to show that $s_n - s_{n+1} > 0$. Therefore $\{s_n\}$ is a monotone sequence.

(c) Use an appropriate theorem to show that $\{s_n\}$ is convergent and call the limit γ . With the aid of Maple or any other mathematical software find the value of γ to five decimal place of accuracy. Find the n that gives such estimate.