Department of Mathematics
Christopher Newport University

Math 355-01 **Complex Variables** *Spring Term 2007*

Homework # 4

Due Friday Feb.16.07

1. Apply the definition (3), Sec.18, of Derivative to give a direct proof that

$$f'(z) = \frac{1}{(1+z)^2} \qquad \text{when} \quad f(z) = \frac{z}{1+z}$$

2. Use an appropriate theorem to show that $f'(z)$ does not exist at any point:

(a) $f(z) = z + \overline{z}^2, \;\; z \neq 0$ (b) $f(z) = e^x \cos y - i\, e^x \sin y.$

3. Use an appropriate theorem to show that each of the functions is differentiable in the indicated domain then find $f'(z)$.

(a) $f(z) = \sinh x \sin y - i \cosh x \cos y$, for all z

(b) $f(z) = e^{-2\theta} \cos(2\ln r) + i\, e^{-2\theta} \sin(2\ln r)$, for $r > 0,\; 0 < \theta < \pi.$

4. Show that each of these functions is entire and find $f'(z)$.

(a) $f(z) = (2x + x^2 - y^2) + i\,2y\,(x+1)$ (b) $f(z) = -\cos x \sinh y + i \sin x \cosh y$